$$
\begin{aligned}
& v=v_{i}+a t \quad a \equiv \frac{d v}{d t}=\frac{d^{2} x}{d t^{2}} \quad A_{\text {sphere }}=4 \pi r^{2} \\
& x=x_{i}+v_{i} t+\frac{1}{2} a t^{2} \\
& x=r \cos (\theta) \\
& V_{\text {sphere }}=\frac{4}{3} \pi r^{3} \\
& v^{2}=v_{i}^{2}+2 a \Delta x \\
& y=r \sin (\theta) \\
& a_{c}=\frac{v^{2}}{r}=r \omega^{2} \\
& \mathbf{F}=m \mathbf{a} \quad r=\sqrt{x^{2}+y^{2}} \\
& R=\frac{v^{2} \sin (2 \theta)}{g} \\
& \tan (\theta)=\frac{y}{x} \\
& F_{c}=m \frac{v^{2}}{r}=m r \omega^{2} \\
& v \equiv \frac{d x}{d t} \\
& A_{\text {circle }}=\pi r^{2} \\
& F_{f}=\mu F_{n} \\
& \text { Symbol Approximate Value }
\end{aligned}
$$


$\qquad$

Short-answer problems: Do any seven problems. Clearly indicate the problem you wish to skip. Six points each.

1. Draw free-body diagrams for the following two situations:
(a) Paul's jet airplane is taking off from a horizontal runway. Friction is negligible, air drag is not.
(b) Flor is on a bridge and throws a rock straight down towards the water. The rock has just been released.
2. Ken's spaceship, with mass $m=8.0 \times 10^{4} \mathrm{~kg}$, is at rest in deep space. Its thrusters provide a force of 1200 kN . Ken fires the thrusters for 20 s , then coasts for 12 km . How long does it take the spaceship to coast that 12 km ?
3. Vaneza's $4,000 \mathrm{~kg}$ truck is parked on a $15^{\circ}$ slope. What is the frictional force on the truck? The coefficient of static friction between tires and road is 0.90 .
4. A mass ( $m_{1}=4.0 \mathrm{~kg}$ ) is on a frictionless slope at $\theta=15^{\circ}$, connected via a massless string and a massless frictionless pulley to a second mass ( $m_{2}=2.0 \mathrm{~kg}$ ) which is hanging freely, as shown in the figure below. What is the tension in the string?

5. If the ramp in the previous problem actually has a coefficient of kinetic friction $\mu_{k}=0.2$ then what is the acceleration of $m_{1}$ ?
6. Find an expression for the magnitude of the horizontal force $F$ in the figure below for which $m_{1}$ does not slip either up or down along the wedge $m_{2}$. All surfaces are frictionless.

7. A flat highway curve has radius 70 m . If the coefficient of static friction between Brandon's car tires and the pavement is 0.95 , what is the maximum speed he can go around this curve without sliding?
8. A satellite orbiting the moon, very close to the surface of the moon, has an orbital period of $T=110 \mathrm{~min}$. What is the free-fall acceleration on the surface of the moon? (Should you need it, the radius of the moon is about $1,740 \mathrm{~km}$.)

Short-answer problems: Do any seven problems. Clearly indicate the problem you wish to skip. Six points each.

1. Answer: (a) The normal force and gravitational force should be equal; the thrust force must be considerably more than the drag force.


Figure 1: Free-body diagram for jet takeoff
(b) Just gravity! There is no "force of the throw" or anything like that.
2. Answer: The acceleration during the thruster burn is

$$
a=\frac{F}{m}
$$

and the speed after 20 seconds would be

$$
v=v_{o}+a t=\frac{F}{m} t=300 \mathrm{~m} / \mathrm{s}
$$

The time to go 12 km would then be

$$
t=\frac{d}{v}=40 \mathrm{~s}
$$

3. Answer: From a free-body diagram, we can see that the component of gravitational force in the downhill direction must be

$$
F_{d h}=m g \sin \theta
$$

The frictional force must be enough to overcome that gravitational component, so

$$
F_{f}=10.1 \mathrm{kN}
$$



Figure 2: Free-body diagram for a recently-thrown rock

Note that we really don't care about the coefficient of static friction in this case, so long is it is sufficient for the job. We could check that by finding the maximum static friction force:

$$
F_{\max }=\mu m g \cos \theta=34 \mathrm{kN}
$$

That's plenty.
4. Answer: Draw free-body diagrams, use $F=m a$, do algebra... you'll get that the acceleration of the system (and thus of mass $m_{2}$ ) is

$$
a=\frac{m_{2} g-m_{1} g \sin \theta}{m_{1}+m_{2}}=1.58 \mathrm{~m} / \mathrm{s}^{2}
$$

The net force on mass $m_{2}$ must then be

$$
F=m_{2} g-F_{T}=m_{2} a \Longrightarrow F_{T}=m_{2}(g-a)=16.4 \mathrm{~N}
$$

5. Answer:

$$
a=\frac{m_{2} g-m_{1} g \sin \theta-\mu m_{1} g \cos \theta}{m_{1}+m_{2}}=0.314 \mathrm{~m} / \mathrm{s}^{2}
$$

6. Answer: Free-body diagrams are necessary... and this is probably the one to skip on this test! In order for the mass $m_{1}$ to not be accelerating up or down the ramp, the vertical component of the normal force $F_{N} \cos \theta$ must equal the force of gravity $m_{1} g$. This gives us the normal force:

$$
F_{N}=\frac{m_{1} g}{\cos \theta}
$$

The horizontal component of normal force is the thing that accelerates $m_{1}$ :

$$
a=\frac{F_{N} \sin \theta}{m_{1}}=g \tan \theta
$$



Figure 3: Free-body diagram for wedge problem

The applied force $F$ must be enough to accelerate $m_{1}$ at that acceleration, but of course it must also accelerate $m_{2}$.

$$
F=\left(m_{1}+m_{2}\right) a=\left(m_{1}+m_{2}\right) g \tan \theta
$$

Does that make sense? We can check by looking at limits. For example, if $m_{1}$ or $m_{2}$ were very small, they should drop out without making the equation invalid, and they do. If $\theta=90^{\circ}$ then the force should be very large, and it is. If $\theta=0^{\circ}$, any force would be too much, and we get $F=0$ at $\theta=0$. So the answer seems reasonable.
7. Answer: The centripetal force is given by friction,

$$
\begin{gathered}
f_{c}=m \frac{v^{2}}{r}=\mu_{s} m g \\
v=\sqrt{\mu_{s} r g}=25.5 \mathrm{~m} / \mathrm{s}
\end{gathered}
$$

That's just under 60 mph .
8. Answer:

$$
m a=m \frac{v^{2}}{r}=m \frac{(2 \pi r)^{2}}{T^{2} r}=m \frac{4 \pi^{2} r}{T^{2}}
$$

You do need to know the radius of the moon, here.

$$
a=1.58 \mathrm{~m} / \mathrm{s}^{2}
$$

