

$$v = v_i + at$$

$$F_f = \mu F_n$$

$$\mathbf{p}_i = \mathbf{p}_f$$

$$x = x_i + v_i t + \frac{1}{2} at^2$$

$$\mathbf{F}_s = -k\Delta\mathbf{s}$$

$$\mathbf{J} \equiv \int_{t_1}^{t_2} \mathbf{F} dt = \Delta\mathbf{p}$$

$$v^2 = v_i^2 + 2a\Delta x$$

$$W \equiv \int_{s_i}^{s_f} \mathbf{F} \cdot d\mathbf{s}$$

$$v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{2i}$$

$$\mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt} = -\nabla U$$

$$P \equiv \frac{W}{t} = \frac{dW}{dt}$$

$$v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}$$

$$x = r \cos(\theta)$$

$$K_{linear} \equiv \frac{1}{2} mv^2$$

$$\mathbf{r}_{CM} = \frac{1}{M} \int \mathbf{r} dm \approx \frac{1}{M} \sum_{i=1}^n m_i \mathbf{r}_i$$

$$y = r \sin(\theta)$$

$$U_g = mgh$$

$$r = \sqrt{x^2 + y^2}$$

$$U_s = \frac{1}{2} k\Delta s^2$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$\tan(\theta) = \frac{y}{x}$$

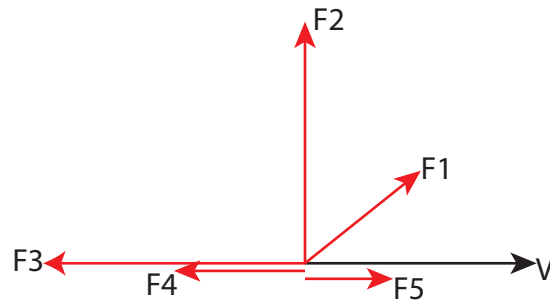
$$\mathbf{p} \equiv m\mathbf{v}$$

$$F_c = m \frac{v^2}{r}$$



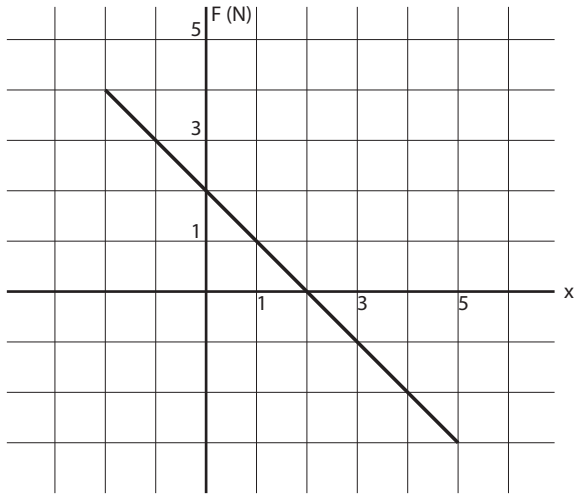
Short-answer problems: Do any seven problems. Clearly indicate the problem you wish to skip. Six points each.

1. A mass  $m$  (not shown) is moving to the right at velocity  $V$  as shown below. Rank, from greatest to least, the power of each force acting on the mass, and explain your ranking.



2. Braedin ( $m = 65 \text{ kg}$ ) is standing on a spring in an elevator which is accelerating upwards at  $a = 3.0 \text{ m/s}^2$ . The spring constant is  $k = 2500 \text{ N/m}$ . By how much is the spring compressed?

3. A 2 kg mass is moving to the right from  $x = -2$  to  $x = 5$ , with an initial velocity of  $v = +5.5$  m/s. While it's moving, force  $F$  acts on it as given in the graph below. What is the velocity of the mass at  $x = 5$ ?

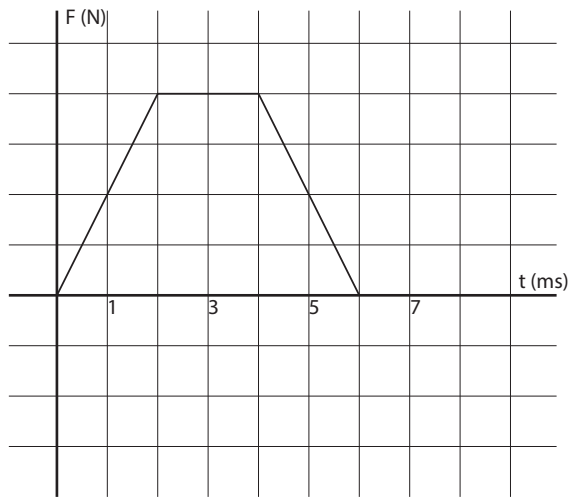


4. When Keenan lands his 15,000 kg jet on an aircraft carrier, the plane's tailhook catches a cable to slow down. The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the plane, how fast was the plane going?

5. A very slippery ice cube slides in a vertical plane around the inside of a smooth, 20 cm diameter horizontal pipe. The ice cube's speed at the bottom of the circle is 3.0 m/s. What is the ice cube's speed at the top?

6. A 20 g ball of clay traveling east at 3.0 m/s collides with a 30 g ball of clay traveling north at 2.0 m/s. What is the velocity of the resulting 50 g ball of clay?

7. A 60 g tennis ball with an initial speed of 32 m/s hits a wall and rebounds with the same speed. The graph below shows the force of the wall on the ball during the collision. What is the value of the maximum force during this collision?



8. Nathaniel fires a bullet of mass  $m$  horizontally into a physics book of mass  $M$  that is at rest on the floor. The book, with embedded bullet, slides a distance  $d$  across the floor. The coefficient of friction between book and floor is  $\mu_k$ . Find an expression for the bullet's initial speed  $v$ .

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1. **Answer:** Power is work over time, which is equivalent to  $\mathbf{F} \cdot \mathbf{v}$ . The most positive power is given by the force with the greatest component in the direction of  $\mathbf{v}$ , so the answer is 1, 5, 2, 4, 3.

2. **Answer:** Draw a free-body diagram: gravity down, spring force up, and

$$F_s - mg = ma$$

You know that  $a = 3.0 \text{ m/s}^2$ , so

$$F_s = m(a + g) = 832 \text{ N}$$

Now it's easy to find the spring stretch:

$$F_s = -kx \implies x = -\frac{F_s}{k} = 0.33 \text{ m}$$

3. **Answer:** The work is the integral of force times distance, and that gives the change in kinetic energy:

$$\int_{x_i}^{x_f} F dx = \frac{1}{2}m(v_f^2 - v_i^2)$$

Of course, integral just means “area under the curve”, so (remember that some of that area is negative)

$$(8 - 4.5) \text{ J} = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$v_f^2 = v_i^2 + \frac{7 \text{ J}}{m} \implies v = 5.81 \text{ m/s}$$

4. **Answer:** Kinetic energy of the plane becomes spring energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \implies v = \sqrt{\frac{kx^2}{m}} = 60 \text{ m/s}$$

5. **Answer:** Use conservation of energy:

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + mgh$$

$$v_f = \sqrt{v_i^2 - 2gh} = 2.25 \text{ m/s}$$

6. **Answer:** The initial eastward momentum is

$$p_e = m_1v_1 = 0.06 \text{ kg m/s}$$

and the initial northward momentum is

$$p_n = m_2v_2 = 0.06 \text{ kg m/s}$$

Well, that's handy. The direction must be  $45^\circ$  east of north. The magnitude of the momentum is

$$p = \sqrt{v_e^2 + v_n^2} = 0.085 \text{ kg m/s}$$

So the magnitude of the velocity is

$$p/m = 1.70 \text{ m/s}$$

7. **Answer:** From the graph, one can calculate the integral and get the impulse from that integral.

$$\int F dt = \Delta(mv)$$

The integral is  $F_{max}\tau$ , where  $\tau = 4 \text{ ms} = 0.004 \text{ s}$ . The change in momentum is  $m\Delta v = 2mv$ . (Remember the change in *velocity* is different than the change in *speed*, which is zero.) Putting this all together we get

$$F_{max} = \frac{2mv}{\tau} = 960 \text{ N}$$

8. **Answer:** There are two parts of the question here: conservation of momentum between bullet and book, and then work/energy between book and floor. Momentum:

$$mv = (m + M)v' \implies v' = \frac{m}{m + M}v$$

Work/Energy:

$$F_f d = \mu_k(m + M)gd = -\frac{1}{2}(m + M)v'^2$$
$$v' = \sqrt{2\mu_kgd} = v \frac{m}{m + M} \implies v = \frac{m + M}{m} \sqrt{2\mu_kgd}$$