$$\mathbf{F}_s = -k\Delta\mathbf{s}$$

$$I_{ring} = MR^2$$

$$I_{disk} = \frac{1}{2}MR^2$$

$$I_{sphere} = \frac{2}{5}MR^2$$

$$I_{rod,CM} = \frac{1}{12}ML^2$$

$$I_{rod,end} = \frac{1}{3}ML^2$$

$$I = I_{CM} + MR^2$$

$$\boldsymbol{\tau} \equiv \mathbf{r} \times \mathbf{F} = rF\sin\theta$$

$$\boldsymbol{\tau} = I\alpha = \frac{d\mathbf{L}}{dt}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$x(t) = A\cos(\omega t + \phi)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\omega_{spring} = \sqrt{\frac{k}{m}}$$

$$E_{SHO} = \frac{1}{2}kA^2$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\mathbf{F}_g = G\frac{m_1 m_2}{r^2}$$

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = G\frac{m}{r^2}$$

$$U_g = -G\frac{m_1 m_2}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

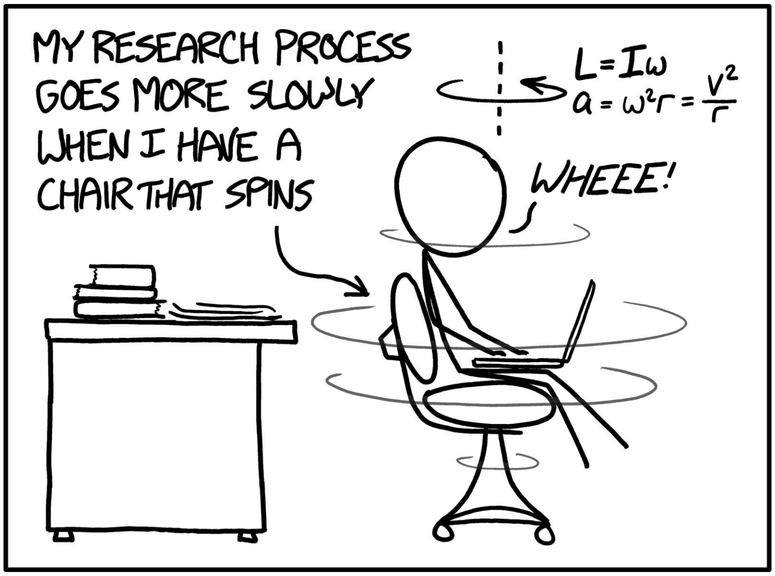
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$F_b = \rho V g$$

$$P = \rho g h$$

$$A_1 v_1 = A_2 v_2$$

| Constant | Symbol | Approximate Value |
|---|---|---|
| Gravitational Constant | $G$ | $6.67 \times 10^{-11}$ Nm$^2$/kg$^2$ |
| Acceleration of Gravity (near Earth) | $g$ | 9.80 m/s$^2$ |
| Mass of Earth | $m_e$ | $5.98 \times 10^{24}$ kg |
| Mass of Sun | $m_s$ | $1.99 \times 10^{30}$ kg |
| Radius of Earth | $r_e$ | $6.37 \times 10^{6}$ m |
| Mass of Mars | $M_M$ | $6.46 \times 10^{23}$ kg |
| Radius of Mars orbit | $R_{mo}$ | $2.28 \times 10^{11}$ m |
| Speed of light in vacuum | $c$ | $2.998 \times 10^{8}$ m/s |
| Standard atmospheric pressure | $P_o$ | $101.3 \times 10^{3}$ Pa |

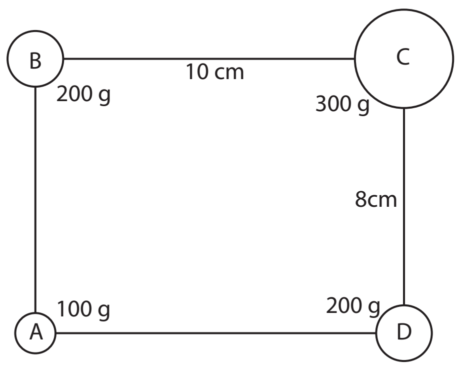

MY RESEARCH PROCESS GOES MORE SLOWLY WHEN I HAVE A CHAIR THAT SPINS
L = Iω
a = ω²r = v²/r
WHEEE!

Name:________________________________________ Score: ______/42

**Physics 204A** **Exam 2 — March 9, 2018**

Short-answer problems: Do any seven problems. Clearly indicate the problem you wish to skip. Six points each.

1. The four small (point) masses shown below are connected by rigid massless rods. Find the moment of inertia about an axis that passes through mass $A$ and is perpendicular to the page.




2. Wei spins a gyroscope by pulling with constant force on a string that is wrapped around the central gyroscope shaft. What is the resulting angular speed $\omega$ of the gyroscope? Here are things that may (or may not) be useful:

   - Rotational inertia $I = 0.0030$ kgm$^2$
   - String length $x = 0.40$ m
   - Constant force $F = 13$ N
   - Gyroscope diameter $x = 8.0$ cm
   - Shaft diameter: $x = 0.50$ cm
   - Gyroscope mass $m = 0.75$ kg
   - Density of seawater $\rho = 1030$ kg/m$^3$

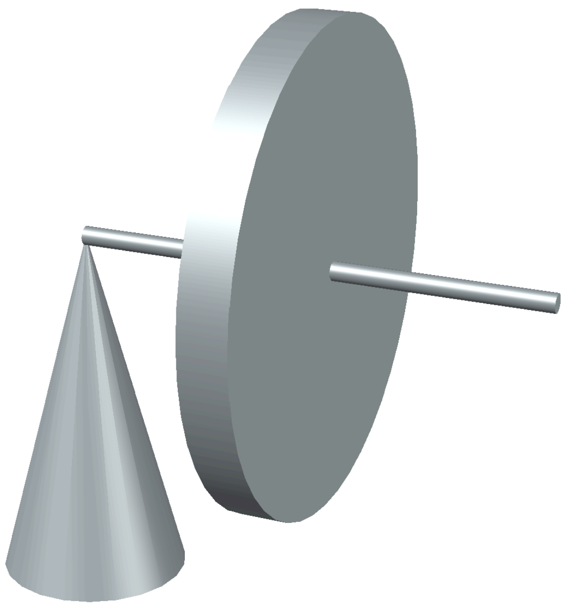
3. For this 3D problem, use the coordinate system where $x$ is positive to the right, $y$ is positive into the exam, and $z$ is positive towards the top of the exam. The gyroscope shown below is spinning clockwise if viewed from $+x$ (looking towards $-x$.) The $-x$ end of the gyroscope shaft is touching the top of the cone, but is free to pivot in any direction.

   (a) What is the direction of $\omega$?

   (b) What is the direction of $L$?

   (c) What is the direction of the gravitational torque $\tau$ on the gyroscope?

   (d) In what direction will the gyroscope move if released?



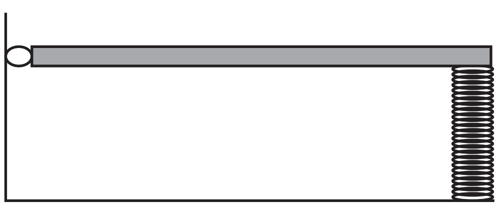
4. A solid disk rolls down a ramp without sliding. It starts at a height $h = 18.0$ cm. It has a mass $m = 2.0$ kg and a radius $r = 5.0$ cm. What is its velocity $v$ at the bottom of the ramp?

5. The charge on a capacitor in a radio transmitter circuit varies according to the equation

$$\frac{d^2Q}{dt^2} = -\frac{Q}{LC}$$

What is the broadcast frequency of this radio transmitter? *Note:* You do not need to know *anything* about electricity to answer this question!

6. A plank of mass $m$ and length $L$ is supported on its left end by a frictionless pivot, and on its right end by a spring of spring constant $k$. What is the period $T$ for *small* oscillations of this plank?



7. Geostationary orbit is an orbit for which the satellite orbits the Earth once every 24 hours. Since the Earth rotates once every 24 hours, this means that the satellite always stays in the same spot overhead. (This is very useful for satellite TV: otherwise you'd have to adjust your satellite dish every couple minutes while watching *The Walking Dead!*) Given the mass of the Earth from the cover of this exam, what is the orbital radius for geostationary orbit?

8. Bubba builds a small personal submarine. It has a volume of 4.2 $m^3$ and a total mass of $m = 4500$ kg. Does this submarine float or sink in seawater? If it floats, what fraction of it is below the surface while it's floating? If it sinks, what is its apparent weight while resting on the bottom of the ocean?

Short-answer problems: Do any seven problems. Clearly indicate the problem you wish to skip. Six points each.

1. **Answer:** The rotational inertia in this case is the sum of the inertias of each "point" mass.

$$I = m_B r_B^2 + m_C r_C^2 + m_D r_D^2$$

$r_B$ and $r_D$ are 8 and 10 cm, respectively; $r_C = \sqrt{r_B^2 + r_D^2}$. Also, $m_B = m_D$, so

$$I = m_B \left(r_B^2 + r_D^2\right) + m_C \left(r_B^2 + r_D^2\right)$$

$$I = (m_B + m_C)\left(r_B^2 + r_D^2\right)$$

$$I = \boxed{0.0082 \text{ kg m}^2}$$

2. **Answer:** Most of those numbers don't really help... I suppose one could figure out the torque from the shaft diameter and the force, the time from kinematics, and use $\tau = I\alpha$ but that's hard. The easy way to do this is to use the work-energy theorem.

$$W = Fx = \frac{1}{2} I \omega^2$$

$$\omega = \sqrt{\frac{2Fx}{I}} = \boxed{59 \text{ rad/s}}$$

3. **Answer:** (a) $-x$
   (b) $-x$
   (c) $+y$
   (d) $+z$

4. **Answer:** Use conservation of energy:

$$E_i = E_f \Longrightarrow mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2$$

$$mgh = \frac{1}{2}\left[mv^2 + \left(\frac{1}{2} mr^2\right)\left(\frac{v}{r}\right)^2\right]$$

$$mgh = \frac{m}{2}\left[v^2 + \frac{1}{2} v^2\right]$$

$$2gh = \frac{3}{2} v^2$$

$$v = \sqrt{\frac{4gh}{3}} = \boxed{1.53 \text{ m/s}}$$

5. **Answer:** This equation is in the form

$$\frac{d^2\square}{dt^2} = -\bigcirc\,\square$$

where $\square = Q$ and $\bigcirc = \frac{1}{LC} = \omega^2$. So since it's in that 'shape' of equation, we know that $\omega = \sqrt{\frac{1}{LC}}$ and

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi\sqrt{LC}}}$$

6. **Answer:** Use $\tau = I\alpha$, try to get things into simple harmonic form. For a plank rotating about one end, $I = \frac{1}{3}mL^2$. At equilibrium, the gravitational torque is countered by the spring torque, so just look at deviations from equilibrium and ignore gravity. $F = -kx$ for a spring, so the spring torque is $-kxL$ for small displacements (small enough that the spring is still at 90° to the board.) Putting things in terms of angles, $\theta = \frac{x}{L}$ so $x = L\theta$ and

$$\tau = I\alpha = -kL^2\theta$$

$$\alpha = \frac{d^2\theta}{dt^2} = -\frac{kL^2}{\frac{1}{3}mL^2}\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$$

So $\omega = \sqrt{\frac{3k}{m}}$ and the period is

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{3k}{m}}}$$

7. **Answer:**

$$m\frac{v^2}{r} = G\frac{Mm}{r^2}$$

$$v = \frac{2\pi r}{T} \Longrightarrow m\frac{4\pi^2 r^2}{rT^2} = G\frac{Mm}{r^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2} \Longrightarrow \boxed{r = 42.3 \times 10^3 \text{ km}}$$

8. **Answer:** Well. . . the mass of 4.2 $\mathrm{m}^3$ of seawater is 4,326 kg. Bubba's submarine weighs more than that, so it sinks. When it's on the bottom, the buoyant force on it is equal to the weight of 4,326 kg of water, so the apparent weight is

$$w = \Delta mg = \boxed{1700 \text{ N}}$$