

Common Substitutions and Approximations

Trig substitutions:

$$\sin^2 x + \cos^2 x = 1$$

$$e^{ix} = \cos x + i \sin x$$

$$\sin x = \frac{1}{2i} (e^{ix} - e^{-ix}) = \cos \left(\frac{\pi}{2} - x \right)$$

$$\cos x = \frac{1}{2} (e^{ix} + e^{-ix}) = \sin \left(\frac{\pi}{2} - x \right)$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y \implies \sin 2x = 2 \sin x \cos x$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y \implies \cos 2x = \cos^2 x - \sin^2 x$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

Approximations:

All of the following series approximations can be derived from Taylor's series:

$$f(x + \epsilon) = f(x) + \epsilon f'(x) + \frac{\epsilon^2}{2!} f''(x) + \frac{\epsilon^3}{3!} f'''(x) \dots$$

So for $|\epsilon| \ll |x|$,

$$f(x + \epsilon) \approx f(x) + \epsilon \frac{d}{dx} f(x).$$

Note: all angles must be in radians!

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots \quad \left(x^2 < \frac{\pi^2}{4} \right)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 \dots \quad (y^2 < x^2)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$$

$$\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 \dots \quad (|x| < 1)$$

So for $|x| \ll 1$

$$\sin x \approx x$$

$$\tan x \approx x$$

$$\cos x \approx 1$$

$$(1 \pm x)^n \approx 1 \pm nx$$

$$e^x \approx 1 + x$$

$$\ln(1 + x) \approx x$$

Stirling's approximation:

$$\ln(K!) \approx K(\ln K - 1)$$

Greek Alphabet

<i>alpha</i>	α	A
<i>beta</i>	β	B
<i>gamma</i>	γ	Γ
<i>delta</i>	δ	Δ
<i>epsilon</i>	ϵ, ε	E
<i>zeta</i>	ζ	Z
<i>eta</i>	η	H
<i>theta</i>	θ, ϑ	Θ
<i>iota</i>	ι	I
<i>kappa</i>	κ	K
<i>lambda</i>	λ	Λ
<i>mu</i>	μ	M
<i>nu</i>	ν	N
<i>xi</i>	ξ	Ξ
<i>omicron</i>	\omicron	O
<i>pi</i>	π, ϖ	Π
<i>rho</i>	ρ, ϱ	P
<i>sigma</i>	σ, ς	Σ
<i>tau</i>	τ	T
<i>upsilon</i>	υ	Υ
<i>phi</i>	ϕ, φ	Φ
<i>chi</i>	χ	X
<i>psi</i>	ψ	Ψ
<i>omega</i>	ω	Ω