Rocket Cars

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Introduction

The equation for the final velocity of a rocket is

$$v_f - v_i = v_e \ln\left(\frac{m_i}{m_f}\right) \tag{1}$$

where v_f, v_i, and v_e are the final, initial, and exhaust velocities, respectively, and m is the mass. This is derived in our textbook on pp. 277–279. In this lab, we will be attempting to verify this equation by measuring the velocities and masses of CO_2-powered cars at the annual ASME CO_2 Car Race.

We will measure the velocity of each car as it passes through two laser beams. By measuring time it takes between blockages of the beams, and knowing the distance between the two beams, we can calculate the average velocity of the car over that distance. Since each car is launched by piercing the CO_2 cylinder to a constant depth with the same firing pin, we can be reasonably certain that v_e is the same for each car.

This measurement method does not tell us the *final* velocity v_f, of course; nor do we know the exhaust velocity v_e. For that matter, we don't know that v_e is a constant for the duration of a car's run, or that the mass of the car as it passes through the speed trap is equal to the final mass. (If the CO_2 cartridge had not completely discharged, it wouldn't be.) Even more worrisome, we are measuring velocities of many different cars, each of which undoubtedly has a different amount of friction and air resistance. However, if these problems are negligible, then a plot of v_f *vs.* $\ln(m_i/m_f)$ will be linear. We will try that, and hope for the best.

Analysis

One way of checking whether differences in friction between the cars had a significant effect would be to plot v_f *vs.* m, and see if the resulting graph follows the relationship we would expect. From $F = ma$ and $v = \sqrt{2ax}$ we obtain

$$v = \sqrt{\frac{2Fx}{m}} \tag{2}$$

So we would expect that v_f would be proportional to $m_f^{-1/2}$. Figure 1 shows a plot of v_f *vs.* m_f with a curve fit to an equation of the form $v = Am^B$. For this data set, $A = 1700 \pm 200$ and $B = -1.01 \pm 0.03$. This is not the expected result!

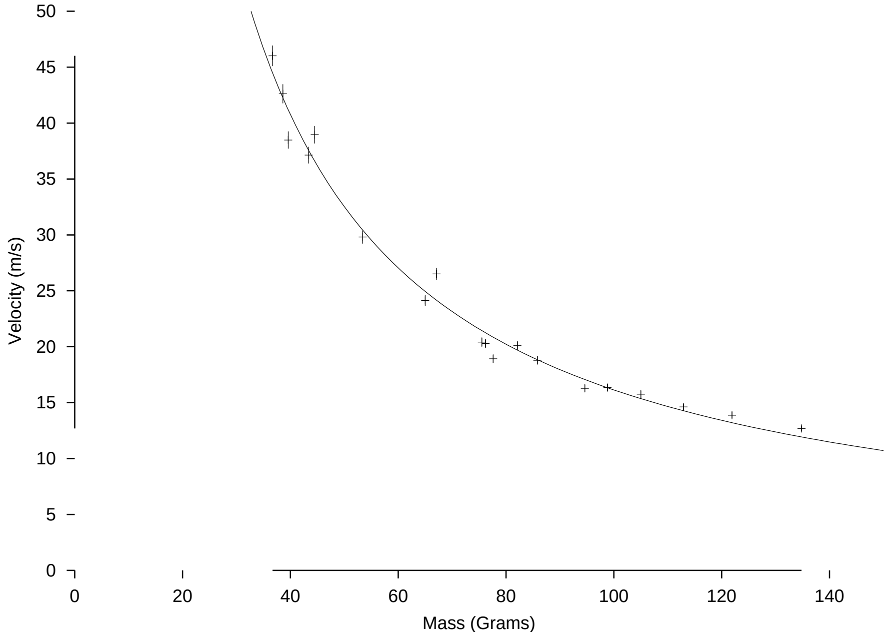

Figure 1: Dependence of velocity on final mass

The most likely reason for this is that the force supplied by the CO_2 cartridge is not constant. A more reasonable model would be an exponential force:

$$F(t) = F_o e^{-t/\tau} \tag{3}$$

so then

$$a(t) = \frac{F_o}{m(t)} e^{-t/\tau} \ .$$

Assuming that the mass is roughly constant (a good assumption on all but the lightest cars in our data) we can integrate this to get velocity:

$$v(t) = \frac{F_o}{m} \int_0^t a \, dt = \frac{F_o \tau}{m} \left(1 - e^{-t/\tau}\right) \tag{4}$$

If the velocity is measured at some time $t \gg \tau$, then $v_f \propto \frac{1}{m}$.

As can be seen in figure 1, there is some variation about the expected curve (probably due to variations in friction between different cars) but mass is the most significant variable, and the data is consistent with equation 4 assuming

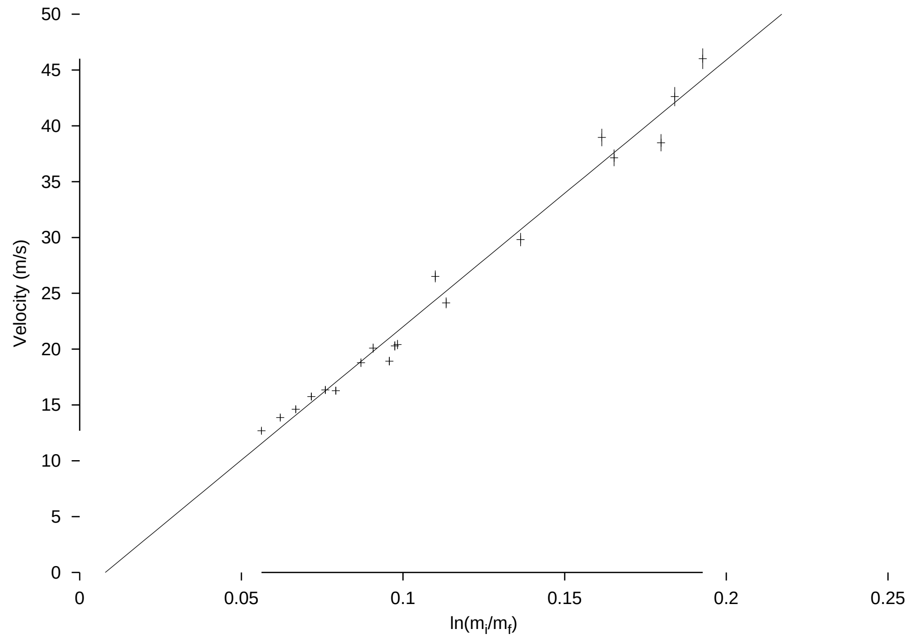

Figure 2: Linear relationship verifying equation 1

that the CO_2 cartridges were largely discharged by the time the cars reached the speed trap.

Figure 2 shows the linear relationship between velocity and natural log of the mass ratio. The linearity of this plot is a strong indicator that equation 1 is accurate, but the derivation of equation 1 assumes a constant value of v_e and thus a constant force. Figure 1 shows that this is not the case! Since the analysis of our data using equation 4 indicates that the CO_2 cartridges were largely discharged before the velocity was measured, we can assume that the slope of the curve fit in figure 2 is the *average* value of v_e. That value, 239 ± 7 m/s, seems reasonable.

Since the initial velocity of each car was zero, one would expect that the intercept in figure 2 would also be zero: The observed value of -1.9 ± 0.9 m/s is a little bit troubling. This offset is likely due to the fact that the mass of the cars—particularly the faster and lighter cars on the right side of the plot—is *not* approximately constant. This would give them a higher than expected velocity, which would shift the intercept down.

Note: Curve fits were done using gnuplot. The uncertainties reported are the asymptotic standard errors given by gnuplot's default fit routine.

Conclusions

We've shown here that the rocket velocity equation *does* seem to describe the velocity of the ASME CO_2 cars, as long as the measurement is made some time after the bulk of the thrust is used up. We've also measured the *average* exhaust velocity of the CO_2 cylinder.

The overly simplistic original model (constant v_e) turns out to be inappropriate for this system; but we've derived a better model (equation 4) that seems to fit the data very well.

It would be interesting to figure out some way of measuring the velocity of the cars throughout the run, to see how well they fit equation 4 over the entire run, rather than just in the limited end-of-run case.