A boiling heat transfer paradox

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An instructive experiment for observing the Leidenfrost phenomenon is presented. The experiment, suitable for an undergraduate experimental course, consists of introducing a copper body at room temperature into liquid nitrogen and observing its temperature history. The experiment is then repeated with the body covered by a thermal insulating material, observing that the body reaches thermal equilibrium much more rapidly in the second case. This apparent paradox greatly motivates the students, who need to understand the different regimes of boiling heat transfer to resolve it. The paper also contains an approximate method to determine the insulator thickness that gives the minimum cooling period.

I. INTRODUCTION

Boiling heat transfer is widely used in engineering applications and is often encountered in physics laboratories. The research in this area began with the works of H. Boerhaave[1] and J. G. Leidenfrost.[2] In 1756 the latter published "A Tract About Some Qualities of Common Water," where he presented some experiments consisting of letting small drops of water evaporate on hot iron surfaces. He found that when the surface was glowing, it took more than 30 s for the drops to evaporate, but when the surface became cooler it took only about 9 or 10 s. Such a paradoxical effect is due to the existence, in the first case, of a vapor film between the drop and the iron which greatly reduces the heat transport and hence retards the evaporation. The experiment has been repeated more recently by different authors[3,4] and similar effects have been observed in a variety of situations, some of which are described in Ref. 1.

Heat transfer involving vapor formation at a vertically oriented heated wall is usually divided into different boiling regimes according to the manner in which vapor is generated. Consider a stagnant liquid in thermodynamic equilibrium with its vapor at a given pressure. The temperature of the liquid will henceforth be referred to as saturation temperature. If the wall temperature is raised above that of the liquid, a first regime will be observed for small wall–liquid temperature difference in which no vapor is generated at the wall. The heat flux is transported by superheated liquid which rises to the free liquid–gas interface, because of buoyancy forces, where it evaporates (see Fig. 1, regime I).

When the wall temperature is increased bubbles are formed at the wall that depart and rise through the liquid. This mechanism of heat transfer is called "nucleate boiling" and is characterized by a steep increase of heat flux with the wall superheat as can be seen in Fig. 1 (regime II). The nucleate boiling regime exists up to a point called the maximum-heat-flux, critical-heat-flux, or "burnout" point (point A in Fig. 1). The name "burnout" comes from the fact that when the heat flux is controlled and is raised above this point, the wall temperature jumps to point B in Fig. 1, which usually corresponds to thousands of degrees. These temperatures are above the melting point of the commonly used heater materials and thus the heater "burns out."

The wall temperature jump appears because a vapor film completely covers the heater and "insulates" it. This heat transfer regime is called "film boiling" (regime IV).

If the wall temperature is now decreased the temperature difference decreases to a point called "minimum-heat-flux" or Leidenfrost point (C, Fig. 1), where the vapor film is no longer stable. If the controlled variable is the heat flux the heater will experience a sudden decrease in temperature, corresponding to a jump to point D in the figure. If the independent variable is the wall temperature, a third regime is encountered between the maximum and minimum heat fluxes in which both nucleate and film boiling coexist alternately. This regime is called transition boiling (region III) and was apparently first described by Drew and Mueller.[5]

The curve displayed in Fig. 1 is called the pool-boiling or Nukiyama characteristic curve, in honor of S. Nukiyama[6] who first obtained it after carrying out experiments with an electrically heated platinum wire immersed in water at saturation temperature. The curve is usually shown as log–log because of the wide ranges of variation of temperature and heat flux.

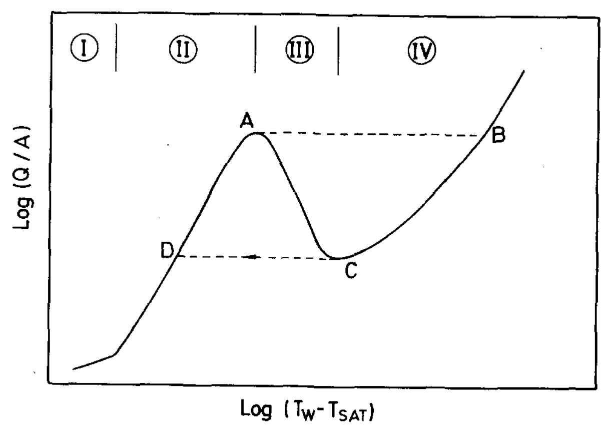

Fig. 1. Typical pool boiling curve showing the different regimes of heat transfer: I: natural convection, II: nucleate boiling, III: transition boiling, and IV: film boiling.

II. EXPERIMENTAL METHOD

One of the simplest ways of measuring the characteristic boiling curve consists of introducing a hot body into the liquid at its saturation temperature. If the temperature difference between the body and the liquid is high enough, the film boiling regime IV will be established.

The body will cool down, rather slowly due to the low heat transfer rates, and Leidenfrost point C will be reached. The vapor film will then break off while the heat flux progressively increases as transition boiling regime III is established.

The body will cool further and the nucleate boiling regime II will be encountered. After this the heat transfer will be accomplished by natural convection currents (regime I) and finally thermal equilibrium will be attained.

From a record of the temperature history the heat flux may be calculated, at least approximately, and thus the boiling curve may be constructed by the following procedure.

It is assumed that the temperature gradients within the body are small. This is a good approximation if the Biot number

$$\mathrm{Bi} = hL/K \tag{1}$$

is small[7] (typically around or below unity), where L is the body's characteristic length, K is the thermal conductivity of the material, and h is the surface heat-transfer coefficient, which is defined as

$$h = \frac{Q/A}{T_W - T_{\mathrm{SAT}}}, \tag{2}$$

where Q is the heat transferred from the body to the liquid per unit time, A is the surface area, T_W is the wall temperature, and T_{SAT} is the saturation temperature of the liquid at the working pressure.

The smallness of the Biot number indicates that the temperature is nearly uniform within the body, corresponding to a situation in which the thermal resistance of the body, measured by L/K, is small compared with the resistance for the heat to flow to the fluid $1/h$. In such a case the temperature difference between any two points of the body is small compared with the difference between the surface and the fluid temperatures.

The problem is that the surface heat-transfer coefficient is not known *a priori* and does not even remain constant throughout the experiment. It would not depend on wall temperature if there existed a linear relationship between Q/A and $T_W - T_{\mathrm{SAT}}$, which is certainly not the case in boiling heat transfer as can be seen from Fig. 1.

Nonetheless it is assumed that the Biot number is small during the cool down; it will be calculated *a posteriori* to assess the validity of this assumption.

Under this hypothesis, the temperature at any point of the body is approximately equal to the wall temperature and to the mean temperature:

$$T = T_W = \overline{T}, \tag{3}$$

where the mean temperature is defined as

$$\overline{T} = \int T\,dV \bigg/ \int dV \tag{4}$$

in which dV is the element of volume and the integrals should be performed over the entire body.

It is thus possible to write the heat balance equation:

$$\frac{Q}{A} = \frac{V\rho c}{A}\frac{dT}{dt}, \tag{5}$$

where V is the volume of the body, ρ is its density, c the specific heat (temperature-dependent), and t the time.

Therefore, a calculation can be performed to obtain the instantaneous heat flux per unit area measuring the temperature history and knowing the physical properties and geometrical parameters of the body. The plot of this quantity against the instantaneous temperature gives the characteristic boiling-curve of the wall–liquid pair under analysis.

The experiment just described is quite classical. It has been performed by Merte and Clark[8] to obtain boiling curves under different gravity conditions and has been reported more recently by Listerman *et al.*[9] as an intermediate level undergraduate laboratory experiment.

The main experiment reported here consists of recording the temperature history of the same body but covered with a thin layer of a thermal insulating material. Again, energy conservation requires that the heat transferred from the body to the insulator equal the heat transferred to the fluid plus the variation of the internal energy of the insulator.

If the variation of the energy stored in the insulator can be neglected and the insulator thickness e is very small compared with the characteristic length of the body, the equation of conduction of heat in the thin thermal insulating layer is reduced to that of heat conduction in a slab:[10]

$$Q/A = \int_{T_b}^{T_a} K_I/e\,dT, \tag{6}$$

where K_I is the thermal conductivity of the insulating material and T_a and T_b are the temperatures at the inner and outer surface of the insulating layer, respectively. If the body covered by the thin insulating layer is a sphere of radius r_a, and r_b ($= r_a + e$) is the external radius of the spherical shell formed by the insulator the heat transfer rate is given by[11]

$$Q\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 4\pi \int_{T_b}^{T_a} K_I\,dT = 4\pi\overline{K}_I(T_a - T_b), \tag{7}$$

where $\overline{K}_I$ is the mean conductivity over the range of temperature from T_b to T_a and the last equation is its definition.

If the temperature of the body is measured, the rate of heat flowing from the hot body may be calculated with Eq. (5). Equations (6) or (7) may then be used to calculate the temperature at the outer surface of the insulator assuming that the temperature at the inner surface equals that of the body.

III. EXPERIMENTAL DETAILS

In the experiment presented here the body was a copper sphere of 25.4 mm in diameter and the working fluid was liquid nitrogen at atmospheric pressure. A chromel–constantan thermocouple was located at the center of the sphere. The chromel–constantan pair was selected because it provides a large voltage compared with other pairs and thus the resolution of the temperature measurement is increased, which is important when the heat flux is to be obtained. The thermocouple was inserted through a 1.5-mm-diam hole and a small amount of solder was added to ensure a good thermal contact. The reference junction was placed in an ice bath and the resulting voltage was measured with a Hewlett-Packard 3455 digital voltometer with an IEEE-488 interface. The digital signal was recorded by an Apple II + computer. Data were acquired at a rate of 1 reading every 0.185 s and stored on magnetic media for off-line analysis.

To insulate the sphere the method favored by long-distance swimmers was used: a layer of grease was applied. Instead of lanolin, Apiezon type-*N* grease was used. This was selected because it has a known thermal conductivity,[12] which is three orders of magnitude smaller than that of copper.[13] The thickness of the grease layer was calculated by measuring the applied volume and assuming the uniformity of the layer. While the uniformity was checked visually no anomalous vapor generation at any location was noticed, an observation made possible because the experiments were performed in a transparent Dewar vessel.

The experiment may be further simplified if it is to be performed in an undergraduate physics laboratory. The results presented below may be obtained replacing the grease by a covering made of paper or teflon tape. However, in this case the results should be considered as being of a qualitative nature because heat transfer across the tape is greatly influenced by the air trapped in between. Another interesting variation is to measure the derivative of the temperature signal with an analog circuit and to send this signal and that of the temperature itself to an *X-Y* recorder. Thus the Nukiyama curve is automatically obtained. Nevertheless the plot is again not of quantitative use because the variation of the specific heat with temperature is not taken into account.

In the present work a fourth-order polynomial was fitted to the copper-specific-heat versus temperature data[14] to calculate the heat transfer. The temperature derivative was computed numerically using a three-point scheme with variable step.[15] This algorithm provides a smooth curve and does not flatten the peak-heat-flux, because the time step selected by the program is the smallest one when the temperature is rapidly varying and is larger otherwise.

The validity of the assumption concerning the temperature gradients within the body may be assessed by placing more thermocouples at different locations in the body. In particular, a thermocouple may be positioned near the surface and connected in opposition with the centered one.[8]

IV. EXPERIMENTAL RESULTS

Results of the measurements carried out with the bare sphere are shown in Figs. 2 and 3. In Fig. 2(a), curve (I), the temperature-versus-time plot shows the typical form for this kind of experiment, while Fig. 2(b) shows the evolution of the heat flux. During the first part of the cooling film boiling regime IV exists and the positive concavity indicates that the heat flux is diminishing as the temperature is decreasing. The change in concavity (at approximately 185 s) corresponds to minimum-heat-flux point C and indicates the beginning of the transition-boiling regime III. The second inflection point (at approximately 194 s) corresponds to the peak-heat-flux at A. From that moment on nucleate-boiling regime II is established and finally thermal equilibrium is attained. The natural convection regime I could not be distinguished in the present experiments. The characteristic pool-boiling curve for liquid nitrogen is constructed by plotting the heat flux against the temperature difference between the body and the liquid

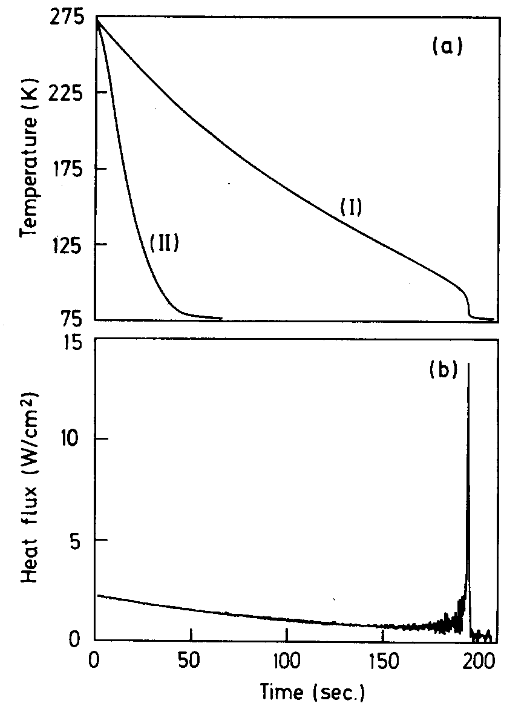

Fig. 2. (a) Experimental cooling curves of a 25.4-mm-diam copper sphere (I) in liquid nitrogen and of the same sphere covered by a 0.3 mm of Apiezon *N* grease (II). (b) Heat flux corresponding to the bare sphere calculated with Eq. (5).

(Fig. 3). This experimental curve is in good agreement with previously reported data.[16]

The Biot number may now be evaluated. To obtain a conservative estimate the values of the heat flux per unit area and wall superheat are taken at the point of the peak heat flux $[(Q/A)_{\rm PHF} = 13.8\ {\rm W/cm^2},\ (T_W - T_{\rm SAT})_{\rm PHF} = 6.2\ {\rm K}]$, thus obtaining a value of 0.51 for the maximum Biot number, based on the sphere radius. Therefore, the approximations already made were reasonable.[17]

Figure 2, curve (II), shows the temperature history of the copper sphere covered by a grease layer 0.3 mm thick. From the graph it becomes clear that the insulated body cools down much more rapidly than the bare one. Defining

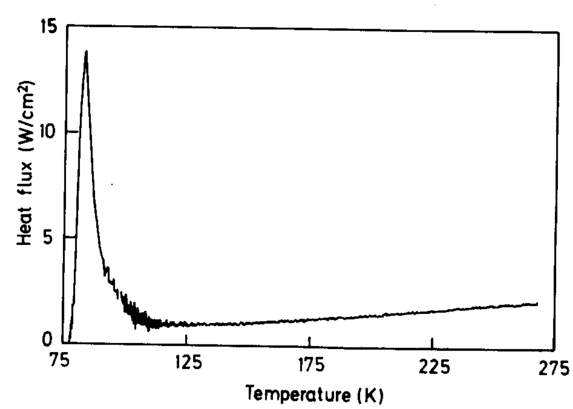

Fig. 3. Experimental pool boiling curve corresponding to the bare copper sphere.

the cooling period as the time required for the body to cool from 273 to 78 K, its value is 196 s for the bare body and only 48 s for the grease-covered body. This is what is herein called the "boiling heat transfer paradox."

The paradox is resolved in terms of the different boiling heat-transfer modes. The wall temperature of the bare sphere is sufficiently high to give rise to the film-boiling regime. This is the cause of a small heat flux which in turn leads to a slow decay of the temperature. Thus the vapor film remains stable for a long period of time. When the film collapses the heat transfer is greatly increased (note that the peak heat-flux is one order of magnitude greater than that of the minimum-heat-flux) and the sphere rapidly reaches the temperature of the nitrogen.

When the insulation is present the temperature measured can no longer be considered as being equal to the surface temperature. Considering the very small thermal conductivity of the grease, a steep temperature gradient exists within it. The surface temperature may be calculated by means of Eqs. (6) or (7) with the heat flux obtained from the copper heat balance, Eq. (5). If the wall temperature is low enough it causes the heat transfer to be in the transition- or nucleate-boiling regime. If this is the case the heat flux is greatly increased and the cooling period is shortened, which resolves the apparent paradox. In fact, the explanation of the experimental results is the same as that of the Boerhaave–Leidenfrost phenomenon, where liquid droplets were found to evaporate very slowly when the heated surface was hot enough to be in the film-boiling regime.

The system may be studied further. Since the temperature drop within the grease depends on the layer thickness it is expected that if the layer is too thin, the surface temperature will not be low enough to cause the transition- or nucleate-boiling regime to exist, thus lengthening the cooling period. On the other hand, once the nucleate boiling exists from the beginning of the quenching a further increase in insulator thickness will again result in an increase of the cooling period.

This behavior is clearly observed in Fig. 4, where the temperature-versus-time curve is shown for different grease thicknesses. Curve (a) corresponds to the bare sphere and the others to insulated-body experiments. Curve (b) was obtained with a layer of 0.025 mm thick. It can be seen clearly that the collapse of the vapor film occurs earlier than in the bare case. With increasing insulation thickness [curve (c): 0.1 mm, (d): 0.2 mm], the temperature drop within the grease is larger, the transition occurs earlier and thus the cooling period is shortened.

Such behavior continues up to the point when the peak-heat-flux is attained from the beginning of the experiment. Further increments of insulation cause the heat transfer to be in the nucleate-boiling regime throughout the cooling, and with thicker layers of grease the initial heat flux decreases due to lower wall temperatures.

Therefore, a critical thickness of insulation may be defined as being that necessary to cause the surface temperature to equal the temperature corresponding to the peak-heat-flux of the bare body when it is at its initial temperature. Using Eq. (7) and with the parameters of the present experiment [inner surface temperature: $T_a = 273$ K, $T_{\mathrm{PHF}} = 83$ K, $Q/A_{\mathrm{PHF}} = 13.8$ W/cm^2, sphere radius: $r_a = 1.27$ cm, grease mean thermal conductivity: $\overline{K} = 0.2$ W/(mK)] it yields a value of external radius of $r_b = 1.298$ cm, which is equivalent to a critical thickness of $e = 0.28$ mm. Approximately this value was used in the experiment of curve (e), $e = 0.3$ mm, in Fig. 4 and should be considered an estimation of the thickness which gives the minimum cooling period. The calculation is not exact since a shorter cooling time may be obtained by beginning in the transition regime and reaching the peak-heat-flux a moment later. This is the case for curve (f) ($e = 0.25$ mm), which begins with a lower cooling rate than that of the previous curve, but then the heat transfer rate is increased and results in a shorter cooling period.

As expected, a thicker layer of grease yields larger cooling periods [curve (g): 0.5 mm, (h): 0.75 mm].

The behavior in the complete range of experienced thicknesses is shown in Fig. 5 in a cooling-period versus thickness-of-insulation plot. The minimum is clearly seen and agrees fairly well with the above-defined critical value.

It should be noted that the present definition of critical insulator thickness has nothing in common with the classical critical radius of insulation which gives the maximum-heat-flux due to an increase in the heat transfer area.[18]

Finally, it is worth noting that, given a pair of fluid-insulator materials, the critical thickness depends on the temperature of the body. The cause of this is that a thicker layer of insulator is needed to force the wall temperature to be at the peak condition if the body temperature is larger.

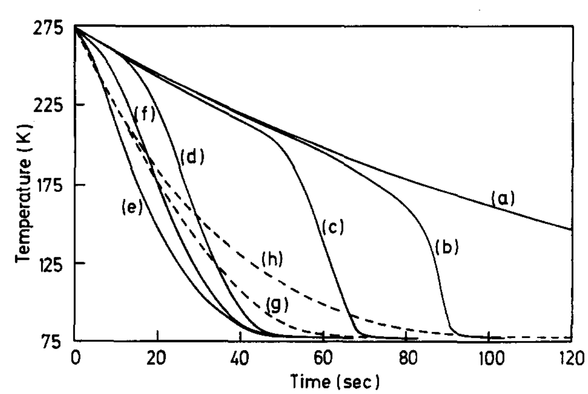

Fig. 4. Cooling curves of the sphere covered by different grease thicknesses [(a): bare, (b): 0.025 mm, (c): 0.1 mm, (d): 0.2 mm, (e): 0.3 mm, (f): 0.25 mm, (g): 0.5 mm, (h): 0.75 mm].

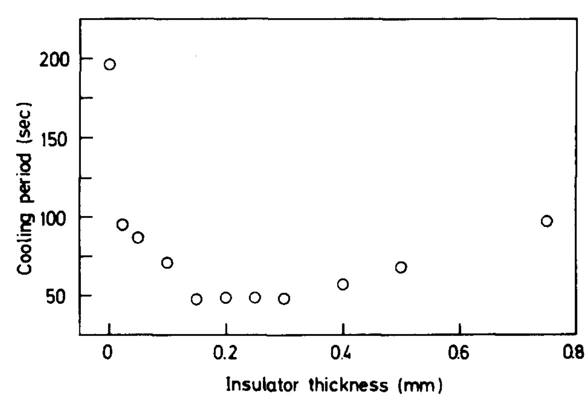

Fig. 5. Time required for the sphere to cool from 273 to 78 K as a function of thickness of insulation. The first point from the left corresponds to the bare-body experiment.

V. CONCLUSIONS

It was shown that adding an insulating layer to an object that is to be cooled in a much colder fluid causes the cooling to be more rapid. This effect was demonstrated with a simple experiment, suitable for an undergraduate experimental course, and is due to the fact that the insulation causes the surface temperature to be lower. This, in turn, causes an anticipated change of boiling heat-transfer mode, from film to transition or nucleate regime, and the subsequent increase in heat flux which shortens the cooling period.

A critical thickness of insulation was defined which approximately gives the thickness of minimum cooling period. It was found to be in good agreement with the experimental data.

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The Liénard–Wiechert potential and the retarded shape of a moving sphere

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The subtleties in the derivation of the retarded Liénard–Wiechert potential for a point charge are stressed by explicitly computing and drawing the retarded shape of a moving sphere. This shape is the effective integration region for the charge density and it is computed, with the aid of the "information collecting sphere," in the limit of vanishing radius (or, equivalently, from the point of view of a remote observer).

I. INTRODUCTION

The retarded scalar potential $\phi(P,T)$, created at time T and position P by a charge density distribution $\rho(\mathbf{r},t)$, is given by[1]

$$\phi(P,T)=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r},T-R/c)}{R}\,dV. \tag{1}$$

Here, R is the retarded distance from P to the point $\mathbf{r}$ at which the source was located at the retarded time $t=T-R/c$.

In the case of a point charge moving with constant velocity v, for given values of P and T the retarded distance R has a single value, say R_0, over the whole charge and the corresponding potential can be written as

$$\phi(P,T)=\frac{1}{4\pi\epsilon_0 R_0}\int\rho\left(\mathbf{r},T-\frac{R}{c}\right)dV. \tag{2}$$

One is then tempted to substitute the total charge q for the integral appearing in the last expression. This, however, would give us an incorrect result. By using the correct value for that integral, namely

$$\int\rho\left(\mathbf{r},T-\frac{R}{c}\right)dV=\frac{q}{(1-\beta\cos\theta)}, \tag{3}$$

one gets the Liénard–Wiechert potential:

