## 5.13 Compton Effect

### Background

Shortly after the discovery of X-rays, it was noted that scattered X-rays had lower energy than unscattered X-rays from the same source. Arthur Compton showed that this was due to the particle nature of the X-rays.[7] (Compton shared the 1927 Nobel Prize for this work.) He applied the laws of conservation of energy and conservation of momentum to the collision between an X-ray and a free (or approximately free) electron, and predicted that the energy of the scattered radiation would depend on the angle.

In this experiment, we will be using $\gamma$ rays instead of X-rays. It is easy to produce a nearly monoenergetic beam of $\gamma$ rays, and the energy measurement is somewhat easier for $\gamma$ rays than for X-rays with the equipment we have available.

### Mathematics

Conservation of momentum tells us that the final momentum of the $\gamma$ ray ($\mathbf{p}_2$) and electron ($\mathbf{p}_e$) is equal to the initial momentum of the $\gamma$ ray ($\mathbf{p}_1$).

$$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_e$$

This, with the law of cosines, leads to

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta \ . \tag{5.13}$$

Energy is also conserved:

$$E_1 + E_o = E_2 + \sqrt{E_o^2 + p_e^2c^2} \tag{5.14}$$

where $E_o = m_ec^2$ is the rest energy of the electron, and $E_1$ and $E_2$ are the initial and scattered energies of the $\gamma$ ray, respectively. Rearranging and squaring both sides of 5.14 gives us

$$E_o^2 + 2E_o(E_1 - E_2) + (E_1 - E_2)^2 = E_o^2 + p_e^2c^2 \ .$$

Substitute 5.13 to obtain

$$2E_o(E_1 - E_2) + (E_1 - E_2)^2 = c^2[p_1^2 + p_2^2 - 2p_1p_2\cos\theta] \ .$$

Now make the substitution $E = pc$, and after a bit more algebra we get Compton's equation in a convenient form for energy measurements:

$$\frac{1}{E_2} = \frac{1}{E_o}(1 - \cos\theta) + \frac{1}{E_1} \ . \tag{5.15}$$

## Experiment

You will be using a sodium iodide detector and a multi-channel analyzer (MCA) to measure the energy of $\gamma$ rays in this experiment. The NaI crystal scintillates when struck by radiation. The amount of light is proportional to the energy of the incident radiation. This burst of light is measured —and amplified— by a photomultiplier tube (PMT). After further amplification, an electrical pulse proportional to the original $\gamma$ ray energy arrives at the multi-channel analyzer, which generates a plot of counts vs. energy and allows us to thus measure the energy of the scattered $\gamma$ rays.

Given a longer lab period, your first task would be to calibrate the MCA. You would do this by observing the channel number at which various $\gamma$ emitters with known energies produced peaks, calculationg a least-squares fit of energy vs. channel number, and so on. Due to time constraints, however, the MCA has been calibrated for you prior to this lab. If you put the cursor on a peak, the software will tell you the energy. *Do not*, of course, adjust the amplifier gain, or this calibration will no longer be valid!

The $\gamma$ rays are generated by $^{137}$Cs, and have $E_1 = 661.65$ keV. A 30-mCi pellet of $^{137}$Cs is housed in the lead house at one end of the lab table. A hole in one lead brick collimates the beam of $\gamma$ rays so that they hit the aluminum target.[13] The NaI detector is housed in a lead "pig", which is a crude but reasonably effective method of making the detector directional.

1. Measure the energy peak for the scattered $\gamma$ rays at as wide a range of angles as possible in the time allowed. Note that it may take some time —especially at the higher angles— to get sufficient counts to accurately locate the peak.

2. If you set $y = \frac{1}{E_2}$ and $x = (1 - \cos\theta)$, then equation 5.15 is of the form $y = mx + b$, with slope $m = \frac{1}{E_o}$ and intercept $b = \frac{1}{E_1}$. Plot accordingly, and determine whether your results support Compton's theory.

[13] Any electron-rich material will do: materials having a low work function are marginally more desirable.