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KATER'S PENDULUM
Physics 258/259

Kater's reversible pendulum is used to complete a precision measurement of the local acceleration of gravity in Storrs Connecticut.

I. INTRODUCTION

Kater's pendulum[1–3] consists of a long bar and two masses which are attached to the bar. The pendulum can oscillate on either of two knife-edge suspension points as shown in Fig.1.

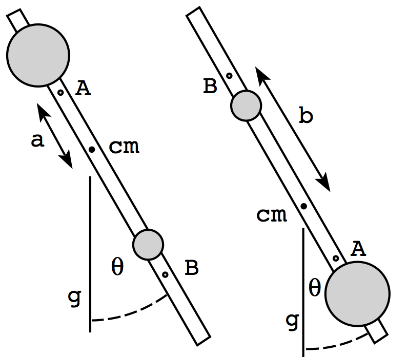

FIG. 1: Kater's pendulum can oscillate about either of the two suspension points A or B. The two distances a and b are from the center of mass (cm) to the pivot points A and B respectively.

For oscillations about point A, the restoring torque on the pendulum of total mass M is

$$N_a = -Mga \sin\theta. \tag{1}$$

Using the parallel axis theorem, the moment of inertia I_a about this pivot point, can be written in terms of the moment about the center of mass I_{cm},

$$I_a = I_{cm} + Ma^2 = M(K^2 + a^2), \tag{2}$$

where $I_{cm} = MK^2$ defines the radius of gyration about the center of mass. Using Newton's second law, the second order differential equation for the angular displacement can be written as

$$I_a \frac{d^2\theta}{dt^2} = -Mga \sin\theta. \tag{3}$$

For small-amplitude oscillations, the solution to Eq. (3) predicts a period of

$$\tau_a = 2\pi\sqrt{\frac{I_a}{Mga}} = 2\pi\sqrt{\frac{K^2 + a^2}{ga}}. \tag{4}$$

A similar analysis of the oscillations about pivot B gives a period of

$$\tau_b = 2\pi\sqrt{\frac{I_b}{Mgb}} = 2\pi\sqrt{\frac{K^2 + b^2}{gb}}. \tag{5}$$

Show that if $\tau_a = \tau_b$, then either $a = b$ or $ab = K^2$. Use the second condition to **show that** if the periods are equal, then they both have the same value of

$$\tau = 2\pi\sqrt{\frac{a + b}{g}}. \tag{6}$$

Equation (6) can then be solved for the acceleration of gravity,

$$g = 4\pi^2\,\frac{(a + b)}{\tau^2}. \tag{7}$$

The quantity $a + b$ is the distance between the knife suspension points, which can be determined very accurately.

II. PROCEDURE

The oscillation period of the pendulum is measured with a photogate and a computer I/O board configured as a counter/timer. When the end of the pendulum interrupts the light path between the source and the receiver, the output of the photogate changes from 5 volts to 0 volts. The computer times the interval between alternate 5 to 0 volt transitions and displays the result on the screen. To start the timing program, run the data-acquisition program. Move your fingers through the photogate a few times to see how it works.

Position the photogate so that the end of the pendulum is centered in the light path when the pendulum is at rest. Start the pendulum oscillating with about a 10° angular amplitude. As the amplitude damps out, the period should decrease slightly. Wait until the period seems to stabilize at a minimum value. Estimate the angular amplitude when you record the period. In order to avoid systematic errors due to the small angle approximation, the angular amplitude should be less than about 5° if the period is to be accurate to four significant figures. Carefully remove the pendulum from its support and replace it so that it oscillates about the other set of knife-edges. Do this gently since these knife-edges are delicate. Be sure that the knife-edge is centered in the retaining-groove. Then measure the period of oscillation. Remove the pendulum, rest it on the blocks on the bench (not on the knife edge assembly!) and measure the distance from the near edge of the large mass to the end of the bar. Then loosen the bolts on the large mass and slide it to a new position. For each position of the large mass, measure the period about both the pivot points A and B. Repeat this for at least eight positions of the large mass. Be sure that you have at least four measurements for which $\tau_a > \tau_b$ and four for which $\tau_a < \tau_b$ and that most of the measurements are for mass positions for which the two periods are reasonably close together. It is a good idea to make a rough plot of period versus mass position as you go along. The distance $a + b$ has been measured in the Physics Department machine shop and will be marked on the pendulum.

III. DATA ANALYSIS

Plot the period versus mass position for oscillations about both A and B on a graph. Draw smooth curves through the two data sets. Where the two curves cross is where $\tau_a = \tau_b$. Use the results to find g. Discuss the error in this value due to the finite time resolution and the non-zero angular amplitude.

The value of the local acceleration of gravity varies with latitude (φ) and elevation above sea level (h) and is often approximated[4,5] as

$$g = g_0 \left(1 + A \sin^2 \varphi - B \sin^2 2\varphi - Ch\right) \tag{8}$$

where $g_0 = 9.78031846\,\mathrm{m/s^2}, A = 5.3024 \times 10^{-3}, B = 5.8 \times 10^{-6}$, and $C = 3 \times 10^{-7}\,\mathrm{m^{-1}}$. Compare this value to the one that you have measured.

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[1] Keith R. Symon, Mechanics, (Addison-Wesley, Reading MA, 1971), 3rd ed., pp. 214-215.

[2] D. Kleppner and R.J. Kolenkow, An Introduction to Mechanics, (McGraw-Hill, New York 1973) pp. 257-257.

[3] J.B. Marion, Classical Dynamics of Particles and Systems, (Sanders, Fort Worth 1995), 4th ed., p 455.

[4] W.A. Heiskanen, and H. Moritz , Physical Geodesy, (W.H. Freeman, San Francisco 1967).

[5] International Association of Geodesy (1971), Geodetic Reference System 1967. Publi. Spéc. n°3 du Bulletin Géodésique, Paris.