Acoustic Bandgap

The Kronig-Penney model for electron wavefunctions in a crystal predicts that there will be "gaps" in the energy: energy regions for which there are no solutions to the Schrödinger equation.[3, 467–470] For very similar reasons, there are forbidden *frequency regions* in the resonance spectra of a corrugated tube. We will use a tube with an internal arrangement of regularly-spaced baffles to investigate the behavior of waves in a periodic lattice. By measuring the resonances as we scan the driving frequency in the tube, we can determine the angular frequencies of the waves allowed in the tube. We can also observe the forbidden region, where there are no allowed resonant frequencies.[1] [2]

Experimental Procedure

1. Begin by writing a LabVIEW program to generate a variable frequency and record the signal amplitude at both ends of the tube for each frequency. It's easiest to do this using the ElvisMX function generator routine for the generation and an Express VI for data collection. Set up the VI to record several thousand samples of input voltage at at least 20 kHz for each frequency. There are built-in routines in LabVIEW (look in Signal Analysis tools) that can tell you the amplitude of a periodic data set. Your program should measure sound amplitude over a range of 10 Hz to 2 kHz at minimum.

2. In addition to the tube resonances we actually care about, there are several other resonances in the system that affect our results. Both the speaker output and the microphone sensitivity depend on frequency. In addition, there is room noise to contend with. These problems can be minimized by looking not at the signal from one microphone but at the *ratio* of the signals from *both* microphones. This gives us the "transmission coefficient" of the tube, if you will. Make sure your LabVIEW program — or data analysis — takes this into account.

3. The gain of the microphone amplifiers can be set quite high. It's easy to set the gain *too* high, in fact. If you look at the microphone signals on an oscilloscope, you may see "clipping" of the signal, where the limited amplifier output prevents the signal from reaching the values it should so the sinusoidal sound wave gets squared off. This clipping will thoroughly mess up your results, so sweep your sound source through a range of frequencies and turn the microphone gain down as needed so

that clipping does not occur at any frequency of interest. You can also use a BNC ‘T’ connector to send the microphone signal to an external oscilloscope in addition to the ELVIS II interface, which allows you to watch for clipping during the frequency sweep. Once you’re satisfied that the gains are set correctly, move on to the next step.

4. Measure frequency response of the bare tube first. (This tube is of smaller outer diameter than the corrugated tubes: the inner diameter, though, is the same as the inner diameter of the spacers in the corrugated tubes.) Plot the response of the tube as a function of frequency.

5. Calculate the wavelength λ of each resonance in the tube, using the frequency, tube length, and anything you might remember from your introductory physics courses. Consideration of question 1 might be worthwhile at this point!

6. Calculate the wavenumber $k = 2\pi/\lambda$ for each resonance mode, and make a plot of angular frequency ω versus k.

7. Next, measure the frequency response of the tube with the evenly-spaced baffles. At low frequencies, the resonances will be nearly the same as for the bare tube, but the resonance frequencies change remarkably after the first few. Using the same technique as in steps 4–6, calculate and graph ω versus k for this tube. (You may assume that each successive peak represents the next possible bare-tube-type mode for the purpose of calculating wavelength.) Overlay your graph on the bare-tube graph for comparison.

8. Make a qualitative comparison to the plot of energy versus k in Tipler (or other text dealing with semiconductor physics.)

9. There is a third tube available: it’s nearly identical to the tube with evenly-spaced baffles, but one baffle is missing. Compare the frequency-response graphs for the two corrugated tubes, and qualitatively relate the results to topics discussed in class. If possible, compare the ω-k graphs for the two corrugated tubes as well.

Questions

1. In introductory physics texts, much ado is made of tubes open at one end versus tubes open at both ends. Is the bare tube (part 4) open at one end or at both ends? This is not a trivial question: look closely at

the relative position of the first few resonance modes and justify your answer.

2. What, if anything, can one deduce from the slope of the plot of ω versus k for the bare tube?

3. Does the speed of sound depend on frequency in any of these tubes? Explain.

4. In a higher-level solid-state physics course one would learn that the first band gap should occur at $k = \pi/a$, where a is the lattice constant of the crystal. Compare and contrast your results with this prediction.

References

[1] Christopher Carr and Roger Yu. Sonic band structure and localized modes in a density-modulated system: Experiment and theory. *American Journal of Physics*, 70(11):1154–1156, November 2002.

[2] David J. Griffiths and Carl A. Steinke. Waves in locally periodic media. *American Journal of Physics*, 69(2):137–154, February 2001.

[3] Paul Allan Tipler and Ralph A. Llewellyn. *Modern Physics.* W. H. Freeman, 4^{th} edition, 2004.