CSUC
Department of Physics
301A Mechanics

Problem Set 1, due Friday, September 7, 2018

I. READING

Please study the class notes on **Dimensional Analysis** by Friday, August 24, and complete the following written problems by Friday, September 7. Read Chapter 1 in your class text through section **1.6** and the following class notes: **Basic Vector Notes** and **Pre-Introduction to Vectors** by September 3. Problems from this chapter will also be found in the next written set. Section **1.7** will be covered in great (!) detail later.

II. PROBLEMS

A. From the Text: *Difficulty Index 1.0*

Problem 1. 1.18 pp. 36
Problem 2. 1.19 pp. 37
Problem 3. 1.20 pp. 37
Problem 4. 1.23 pp. 37

B. From the Class Lectures and Notes: *Difficulty Index 1.0*

Problem 5. A standard exercise in unit conversion.
A system of units often used by mechanical engineers chooses, in addition to the foot and the second, its third fundamental unit as a unit of force (*not* the usual mass!) called the *pound-weight* or "pound" for short. The unit of mass then becomes a derived unit if we are to use a set of units ***coherent*** with Newton's second law in its usual *French* form. This unit of mass is called the *slug*. Express the slug in terms of the fundamental units (foot, lb, sec). Now, compare each of these units to the standard SI unit of like kind. Find the (dimensionless) *numerical value* of the fundamental gravitational constant $G_{English}$ in this system.

Problem 6. A standard exercise in unit definition.
The coefficient of viscosity η is *defined* by the equation $\frac{F}{A} = \eta \frac{dv}{ds}$ where F is the frictional force acting across an area A in a moving fluid, and dv is the difference in velocity parallel to A between two layers of fluid a distance ds apart (measured perpendicular to A - please see the figure below). Find the dimension of η and the units in which it would then be expressed in the SI and {pound, foot ,sec} unit systems. Find the conversion factor between these units.

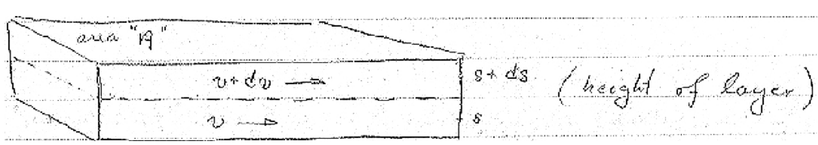

FIG. 1: The geometry of the experiment defining the coefficient of viscosity

Problem 7. Dimensional considerations almost totally solve many problems...like this one !
A fluid flows through a cylindrical pipe of length l and radius a and we wish to model the flow and find the necessary relationships among the descriptive variables. A pressure difference ΔP (force per unit area) causes a flux Φ (volume per second) to flow through the pipe. Let's start by making the natural "model assumption" that ΔP is strictly proportional to l but otherwise depends only on: (a) Φ, (b) on the radius a of the pipe, and (c) on the viscosity η as defined above. We write, then, as our starting model: $\Delta P = l\, f(\Phi, a, \eta)$. Show from dimensional considerations **alone** that it then follows that the unknown function f can be known up to an undetermined dimensionless constant. Show, in fact (!), that ΔP *must* be proportional both to η and to Φ and inversely proportional to a^4.

Problem 8. This is a quintessential "Natural Units" problem. Find Them! *Difficulty Index 2.0*
A mass m is located by its spatial coordinate x at time t. Imagine that its motion is governed by the following *energy* relationship:

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 + \frac{l^2}{2\,m\,x^2} - \frac{k}{x} = E \tag{1}$$

When we measure the world using M.K.S. units, the constants m, l, k, and E take the numerical values $m = 5.976 \times 10^{24}$, $k = 7.928 \times 10^{44}$, $l = 2.661 \times 10^{40}$, $E = -2.6515 \times 10^{33}$.
These values are clearly *ridiculous* and we seek "more natural" units.

a.) Find a *new* set of length, time , and mass (... and so also energy) units (expressed both symbolically in terms of $\{m, l, k\}$ and as M.K.S. values) such that using them our equation assumes the following much simpler form:

$$\frac{1}{2}\left(\frac{dx'}{dt'}\right)^2 + \frac{1}{2\,x'^2} - \frac{1}{x'} = E' \tag{2}$$

where x' and t' are the measured numbers in this new "natural" unit system. Can you discern what these new "unit" quantities really are? (Hint: This is a real problem from a setting YOU know and love!)

b.) Now find the "simple" numerical value of E' in this new unit system. Comment.

Problem 9. Building a 'Hot-Air' Balloon: an estimation exercise *using* natural sizes and scaling. *Difficulty Index 2.0*
Hot air balloons work because the hot air inside the balloon is at the same *pressure* as the cold air outside but weighs *less* because it's less dense. If the balloon started with cold air inside, which we then heated, some of the air would have escaped from the balloon's fixed volume as the heated air inside expanded. Archimedes' law tells us that the buoyant lift we achieve is just the weight of the air that left. Since cold air 'floats' in cold air, each kilogram that leaves the balloon enables us now to lift a kilogram of our *own* payload! Of course, part of our payload is the physical mass of the balloon itself. We now ask: 'under average circumstances, how big must a "hot-air-balloon" be to just lift *itself* ?' You are to estimate the minimum size of a neutrally buoyant balloon. This summer I tried this with .6 mil plastic sheeting, which has a mass density of 15.6 grams for each square meter of surface. Assume your balloon is well approximated as a sphere and that your starting temperature is 300K, which you then heat to 310K inside the balloon. Recall that, at STP ($T = 273.15\ K$, $P = 1\ Atm.$) , air (an ideal gas) has a molar volume of 22.4 liters and a molar mass of 29 grams. Start from the ideal gas law $PV = nRT$ and assume the pressure stays constant at one atmosphere. Notice especially that the surface area (and thus the structural mass) increases as the square of the radius, but the volume (and thus the 'lifting ability') increases as the radius cubed. Thus, for *"big enough"* radius, you are sure to win!! ... but at what radius will that (*approximately*) be?

a.) How big would the balloon have to be to lift **itself** ? (**Trick**: find the right natural units!)

b.) How big would the balloon have to be to lift itself and **you** ? (**Trick**: start from the previous solution and *build* the new solution to the *new* problem out of the old solution! This is, at its essence, a **scaling** problem! So much of Physics works this way. Problems almost always build out of easier problems ... through some simple transformation. In this case, scaling will lead you to a simple cubic equation and our answer is its root. Simple graphing solves it !)

III. DISCUSSION OF THE PROBLEMS

Dimensional analysis is far more than just *getting the units right* . It's primary use is in studying the *scaling* laws implicit in the problem. These are the 'shape relationship' questions of the problem. They have their origin in the form of the underlying elemental physical laws. The brute *algebraic* form of the starting laws actually demands, *of itself*, much of how any answer must behave ... e.g. how the different variables must ultimately sit next to each other in any solution. Along with this is the topic of 'natural units' . Nature has its *own* sizes! We will always find them in the problem itself. Learning to recognize the meaning of 'dimension' and where it comes from, learning to extract the scaling laws implicit in dimensional knowledge, and learning to scale any equation according to the 'natural units' *of* the problem are the lessons of this problem set. You may find these considerations and techniques strange at first - but you will come to love them and see their depth and power. They are oh-so *"Physics-like"* ... and, once learned, there is no looking back!