CSUC
Department of Physics
301A Mechanics
Problem Set 2, due: Monday, October 1, 2018 (Enrico Fermi's Birthday is 9/29/1901! Rome)

I. READING

Read Chapter 2 in your class text completely . Read (or re-read) our class notes **Pre-Introduction to Vectors** , **Pre-Introduction to Determinants** , and **Introduction to Vectors** .

II. PROBLEMS

A. From the Class Text: *Difficulty Index: as marked in text ⋆ symbols.*

Please complete for submission the following text problems from *chapter 2*:
(1.) Page 73 problem *2-7*
(2.) Page 73 problem *2-8*
(3.) Page 74 problem *2-9*
(4.) Page 74 problem *2-10*
(5.) Page 78 problem *2-36*
(6.) Page 80 problem *2-49*

B. From the Class Lectures: "Old Problems you know and love ... or do you?" *Difficulty Index 2.0*

(7.) The old "maximum range" problem. A cannon shoots from on top of a hill of height H. Ignore air resistance altogether and find the modifications to the standard elementary ballistics range problem we all do in elementary mechanics.
a) At what angle should the cannon shoot *now* to achieve maximum range? Comment!
b) Next, find the maximum *range* that the cannon actually achieves at this maximizing angle. We wish to use our answer to address the question: "Does it pay to position artillery on hills ? " Use the US 155 mm howitzer as your comparison. This venerable device saw service from WWII until the present and achieved an impressive muzzle velocity of 1,847 ft/sec. Suppose that H = 100m. Comment!

(8.) The old "SHO and Dry Sliding Friction" problems. In elementary courses everyone studies the simple harmonic oscillator with its linear restoring force $F_{SHO} = -kx$ and also describes friction through the simple model $F_{fric} = (\pm)\mu mg$. Usually these are examined separately. But suppose we put it together now and really *do* try to describe a mass on a table top connected to a spring. What do we get? You are to show that - surprisingly! - the resulting oscillations *remain* isochronous (period independent of amplitude) but that the amplitude now diminishes by a *constant increment* in each half cycle. Show that this is so and find that increment. HINT: You will need to solve the DEq in pieces as it is NOT analytic! This is your *"Let's Solve a Differential Equation"* problem.

(9.) The old "Three Points Determine a Circle" problem. A classic theorem of geometry states that any three distinct points determine a unique circumscribed circle. But how shall we actually find that circle? You will use vector methods. Start by reading the class notes "Vectors" and "Pre-Introduction to Determinants". The action takes place in a plane but that doesn't mean we can't use the third dimension. Consider the picture below. Choose your origin at one of the points (say at point A) and define three vectors $\vec{a} \equiv \vec{AB}$, $\vec{b} \equiv \vec{AC}$, and $\hat{z}$ (this one out of the page). The center, where ever it is, will be located by another vector $\vec{R}$ which it is now our problem to find. The center is defined by the fact that it is equidistant from all points on the circumference. This means we must have *both* $|\vec{R}| = |\vec{a} - \vec{R}|$ and also $|\vec{R}| = |\vec{b} - \vec{R}|$. Use these two equalities plus your knowledge of the "Reciprocal Basis" of $\{\vec{a}, \vec{b}, \hat{z}\}$ to actually *find* $\vec{R}$ and finally show that $|\vec{R}|$ is given by the product of the lengths of the sides of $\Delta\, ABC$, the circumscribed triangle, divided by four times its area.

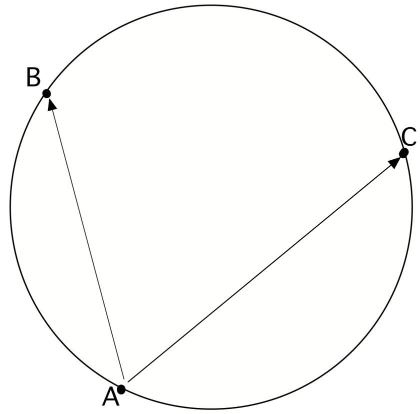

FIG. 1: The geometry of the circumscribed circle.

III. NOTES ON PROBLEM SET 2

This set is designed to present problems which are not conceptually new - but rather a step closer to the true complexity with which we are confronted by the physical world. I have chosen the problems specifically to point out various features which occur commonly.

problem (5) : What a great problem! Every physicist has the "moral duty" to compute their way through this historically important story, which, as is now firmly believed, really did happen. Treat it directly and don't skimp as the answer is interesting. As always, "numerics" are surprisingly tough and messy - something most people "out on the sidewalk" rarely appreciate. This is your *"numerics"* problem.

problem (7) : What could be conceptually simpler than this problem? Unfortunately, a direct solution is fiendishly difficult ! Go ahead and try if you like, but don't blame me for the pain! This one simply will not be bludgeoned into submission. It is characteristic of real problems that a more formal approach is often necessary. As it happens, the simple *algebraic inversion* of one of your equations seems so obvious - but returns an expression you just can't live with ... and can't force ... so this "obvious" strategy utterly fails! Here, however, we can use *implicit differentiation* (a technique you should know from your calculus background already) to excellent effect. The answer is *ridiculously* simple and *oh-so-beautful ...* and should make you a firm believer in natural units! A problem to absolutely hug and love. (N.B. If you prefer, a yet more stylized technique we will introduce and seriously focus on later, but which you may already know, ... namely the use of *Lagrange Multipliers* allows you to phrase this problem as a *constrained extremization* problem. This too, will "knock it dead" ... and in time to come will absolutely become one of your "GO TO" techniques.) This is your *"more advanced techniques"* problem.

problem (8) : This problem is designed to encourage you to "trust" differential equations. Because dry-sliding friction is "non-analytic" (i.e. it has a jump discontinuity upon changing directions of motion) ... you must solve the equation of motion in pieces. Each half-swing restarts the process, so you need only understand one standard "half-swing". So! This is not a difficult problem ... only a little messy ... and so characteristic of what we come up against all the time. This is your *"differential equations"* problem.

problem (9) : All students at this level need (and I mean "really need") to do lots of *heavy lifting* with vectors because facility with vectors is so central to our ability to "do physics". Practice practice practice! The idea of a "reciprocal basis" set of vectors is probably new to you ... though not difficult to grasp. Actually, it forms a pivotal concept all over physics and even in differential geometry ... the mathematical foundation of general relativity. Don't let it scare you. Use it with confidence and pleasure. This is your *"Let's Deal with Vectors"* problem.