CSUC
Department of Physics
301A Mechanics
Problem Set 4, due: Monday, October 29, 2018

I. READING

Read Chapter **4** in your class text completely .

II. PROBLEMS

A. From the Class Text: *Difficulty Index 1.0*

Please complete for submission the following text problems from *chapter 4*:
(1.) Page 150 problem *4-2*
(2.) Page 150 problem *4-4*
(3.) Page 152 problem *4-10*
(4.) Page 152 problem *4-14*
(5.) Page 154 problem *4-28*
(6.) Page 154 problem *4-30*
(7.) Page 156 problem *4-36*
(8.) Page 156 problem *4-37*

B. From the Class Lectures and Notes:

(9.) A very popular model for the potential energy between two atoms in a diatomic molecule is the classic old 'six-twelve' model.

$$V(x) = -\frac{a}{x^6} + \frac{b}{x^{12}} \tag{1}$$

where x is the distance between the atoms and a and b are positive constants which are adjusted for any specific case under study.
a) Find the force.
b) Assume that one of the atoms is very heavy and remains at rest while the other moves along a straight line. Describe the possible motions.
c) Find the equilibrium distance and the period of small oscillations about it if the lighter particle has mass m.

(10.) *Difficulty Index 2.0*
A pendulum mass is constrained to follow the path of a cycloid . We may parametrize its position with a new variable ϕ as

$$x = a(\phi + sin(\phi)\,,\, and\;\; y = a(1 - cos(\phi)). \tag{2}$$

Show that the period of oscillation does **not** depend on the amplitude and has value $\omega = (\frac{g}{4a})^{\frac{1}{2}}$.
A cycloid is one of those "miracle curves" appearing everywhere in physics. Most simply, if we mark a point on a non-slipping rolling wheel and follow its trajectory we observe a cycloid traced out. However, the cycloid is also the unique curve that is its own evolute (and thus its own involute!). This means that if we constrain a pendulum to swing between two cycloid 'cutouts' that the pendulum bob traces out ... yes, you guessed it ... the cycloidal shape of the cutouts! We will also discover that it is the solution of the brachistochrone problem just as here we observe that it is the true solution of the isochrone problem.

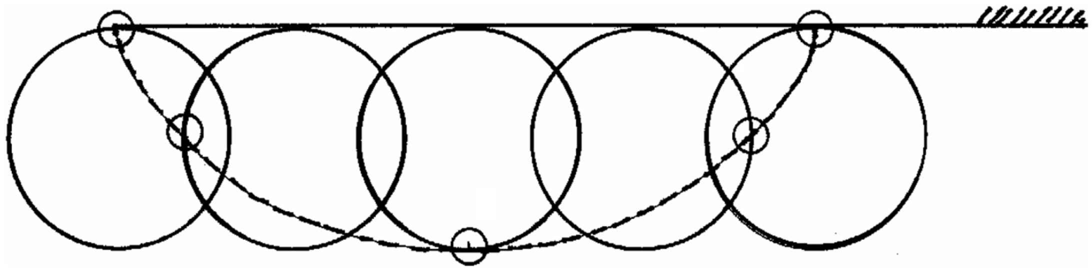

FIG. 1: A cycloid is generated by a wheel rolling on the ceiling.

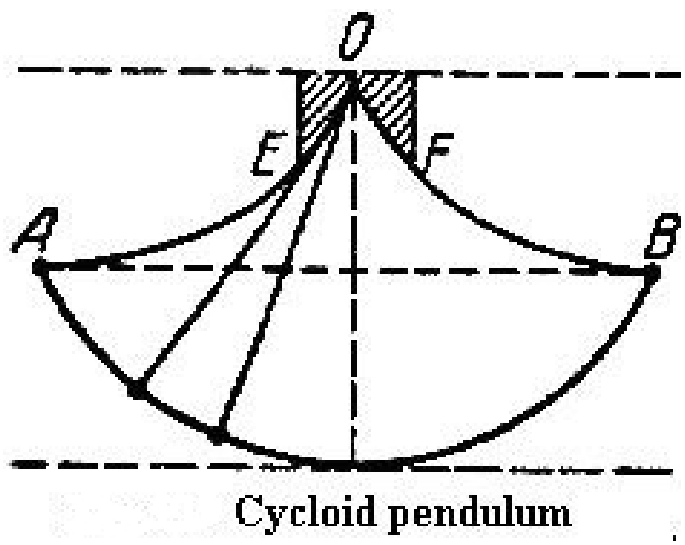

FIG. 2: A cycloid is generated by a pendulum swinging within cycloidal cutouts.