CSUC
Department of Physics
301A Mechanics

Problem Set 6, due: At the end of Finals' Week, December 21, 2018

I. READING

Read Chapter 8 in your class text omitting only the bottom of page 299 (for now) .

Comments on the Problems The following problems should be approached in a somewhat different spirit than the previous sets. These problems are "historical" so to speak. They were first studied rather early-on in the modern Enlightenment immediately following Newton and set the tone for all that has followed. You retrace the path of real pioneers who recognized that they were plumbing the depths of the very inner machinery of the cosmos. They learned how the arguments must fall out ... and so will you. Their conclusions (along with modern descendants seasoned by the staggering revelations of Relativity) are in current use to this hour. In fact, those who have not made this journey can never truly appreciate the structure, mindset and vocabulary of modern science. Indeed, you might easily be said to be on a journey to a foreign country ... a *Community of Scholarship* ... from which you can never truly return. The others will become as strangers to you. One cannot unlearn the truth.

II. PROBLEMS

A. Problems from the Text:

Please complete for submission the following text problems from *chapter 8*: Most of the difficult problems can be addressed merely by an application of the two conservation laws of energy and angular-momentum.

(1.) Page 321 problem *8-3* A really simple problem of deep import. One begins to see *General Relativity* around every corner - nor are you far deceived.

(2.) Page 322 problem *8-13 (difficulty index 1.5)* Once you have completed the text parts of this exercise, please *extend* this problem in the following way. First, find the **Force Law** in Cartesian coordinates and you will discover that the differential equations for the x and y coordinates may be trivially separated from each other. Next, find their simple *general solutions* and you will discover that the orbit **must** be an *ellipse* ... only this time described from the *center* rather than the focus of the orbit.

(3.) Pages 322-323 problems *8-12* and *8-14 (difficulty index 2)* (which form a single problem) This is Newton's classic calculation. He realized Gravity *has to be* inverse square ... because the orbits are observed to be **closed** ! In each case you will argue from the conservation laws and perform a "small expansion" about the stable orbit. *NOTA BENE* ! In our historical perspective, we can now perceive just how important precision astronomical observations have proven to be! They are **crucial**! Any deviation from closed orbits would indicate either a here-to-fore unnoticed perturbation or some necessary correction to our theory.

(4.) Page 323 problem *8-17* A simple problem that introduces a "strange idea". We will understand later where this really comes from.

(5.) Page 323 problem *8-18* Richard Feynman used to predict where those early astronauts would land in the ocean! He could do it with really eye-popping accuracy because he realized that a little knowledge goes a long way. You have to command problems like this ... and somehow trust that "Nature" really does *follow the law.*

(6.) Page 323 problems *8-23 and 8-24 (difficulty index 2)* A **GREAT problem**. Do it sensitively. Extend this problem by showing that the solution may be understood as a *precessing ellipse.* Determine the *angular velocity of precession* and determine whether it is in the same or opposite direction to the orbital angular velocity.

(7.) Page 324 problem *8-29* Not a hard problem!

(8.) Page 324 problem *8-31* You will use this problem later when we study scattering theory. It should remind you of your study of Bohr orbits. Negative energy yields *bound* orbits (or as we learn to say "states") and positive energy values are associated with orbits of unbounded physical size that you will learn to call *scattering* states.

B. Problems from the Class:

(9.) *(difficulty index 2)* It is "oh-so-important" to build up a repertoire of possibilities as to what can happen in Nature. Suppose that we have encountered an attractive central *inverse cube* force viz. :

$$\vec{F}(r) = -\frac{K}{r^3}\,\hat{r} \qquad \textit{where} \quad K > 0$$

a) Discuss the possible motions by means of the "effective potential" technique and find the ranges of the Energy E and Angular Momentum ℓ for each type of motion.

b) Now solve the orbital equations in detail and observe that the solution must be one of the following forms:

$$\frac{1}{r} = A\,cos\{\beta\,(\theta - \theta_o)\} \tag{1}$$

$$\frac{1}{r} = A\,cosh\{\beta\,(\theta - \theta_o)\} \tag{2}$$

$$\frac{1}{r} = A\,sinh\{\beta\,(\theta - \theta_o)\} \tag{3}$$

$$\frac{1}{r} = A\,\{\beta\,(\theta - \theta_o)\} \tag{4}$$

$$\frac{1}{r} = A\,\frac{1}{r_o}\,e^{\pm\beta\,\theta} \tag{5}$$

c) For what values of E and ℓ does each of these cases occur ? Express A and β in terms of E and ℓ and provide a sketch of each of the orbits.