CSUC
Department of Physics
301A Mechanics

Problem Set 7, due: End of Fall Exam Week, December 21, 2018

I. READING

Read Chapter 7 in your class text sections 7.1 - 7.7 omitting 7.4 on your first reading .

II. PROBLEMS

This unit introduces standard introductory "Lagangian" problems. You will very soon come to trust that this approach yields exactly what a detailed Newtonian analysis would ... only with a tiny fraction of the effort! In fact, the Newtonian approach rapidly becomes entirely unmanageable as the problems increase in complexity while the Lagrangian prescription remains rather elementary. You are likely to become rather supercilious of those using *merely Newtonian* methods and will yourselves be transformed into devoted "Analytic-Mechanicians". We will see later that this is completely justified. Indeed, we are now on the *Royal Road* leading straight to the heart of physics. Electrodynamics, Optics, Statistical Mechanics, General Relativity and Quantum Mechanics will *all* emerge in a completely natural manner from this reasoning and the **Unity-of-Physics** will be firmly reestablished.

A. Problems From the Class Text:

Please complete for submission the following text problems from *chapter 7*:

(1.) Page 281 problem *7-3* An Elementary Problem to start with. It is not required, but for those with a hankering for "bragging rights" in the club-house ... prove that ALL the orbits must, in fact, be ellipses.

(2.) Page 283 problem *7-14* A seemingly elementary problem but which can be mined for "oh-so much". You are to extend it by finding the "force of constraint" (i.e. the frictional force at the contact point) by employing the methods from class.

(3.) Page 285 problem *7-29* Lagrangian Physics doesn't care *how* you proceed algebraically ... do it however you like (or however it occurs to you ...). The freedom given you is, at first, shocking and troubling - but you will develop a taste for it.

(4.) Page 286 problem *7-31* Similar comments to those just above apply here too. By the way, in the last step (the small oscillation analysis) I disagree with our Text. **Don't** expand the equations of motion! Rather, expand the Lagrangian itself to 2nd order in small quantities and only then re-generate the SHO equations of motion.

(5.) Page 287 problem *7-35* Same as above. This problem is generalized in problem **(10.)** below.

(6.) Page 288 problem *7-38* A classic problem. Use the "Effective Potential" approach for the SHO analysis.

(7.) Page 288 problem *7-40* Same as above. You should recognize this problem from elementary mechanics. Make sure you compare all your "more involved" results with elementary deductions from long ago. They must, of course, agree!

(8.) Page 288 problem *7-41* A really really surprising problem. I don't like his parameterization. Please describe the *Parabola* via $z = \frac{1}{2R}\rho^2$ where $\rho^2 = x^2 + y^2$. In doing so we not only manifestly preserve the proper dimensionality, but then R also has the correct physical meaning of "radius of curvature" at the vertex.

B. Problems From the Class

9.) A rod of length L and mass M leans up against a smooth wall and slides against the wall and floor without any friction. Set up the equations of motion for the rod assuming that it maintains contact with the wall. If the rod starts from rest at a given angle α , at what angle (if any) will it **leave** the wall? (N.B. You can answer this question more than one way! e.g. examine the 'Normal Force' from the wall . . . or by following the motion of the Center of Mass . . .)

10.) A hoop of mass M and radius R rests horizontally on a smooth table (just as in problem (5.) above . . . think of this as a "continuation problem"). However, . . . this time . . . one point on the circumference is now pinned to the table so that the hoop may swing **freely** in the horizontal plane about that pinned point and a cute little *bug* of mass m crawls at constant speed v_o about the circumference of the hoop.

(a) First, set up the equations of motion for this problem by choosing to view it as a *two coordinate* problem (i.e. two degrees of freedom) with the force that the bug exerts against the hoop to be determined from the condition that the bug moves with constant speed along the rim of the hoop.

(b) Second, set up the equations of motion for this problem by choosing to view it as a *one coordinate* problem (i.e. only one degree of freedom) and with a time dependent constraint embedded in the geometry of the problem at the outset. Show that the two formulations yield equivalent equations of motion for the hoop.

11.) The industrial revolution led to the invention of myriads of clever devices to control the machines of production. The "Flyball Governor" for steam engines and mills is one such device. Two equal *flyballs* each of mass **m** are attached to a vertical rotating shaft by means of four equal hinged arms of length ***l*** joined to the shaft via two hinged sleeves. The upper sleeve is welded to the shaft and turns with it, but the lower one slides up and down freely and has mass **M**. Thus, the balls are free to swing outward and in doing so change the height of the lower sleeve. For this problem, assume that the shaft is turning at a constant angular velocity Ω , and we must find the equation of motion of the system and discuss it via the energy method. This problem is a "Grand Old Nugget" ... *everyone* does this problem! Use your "Physics Intuition" in each step.

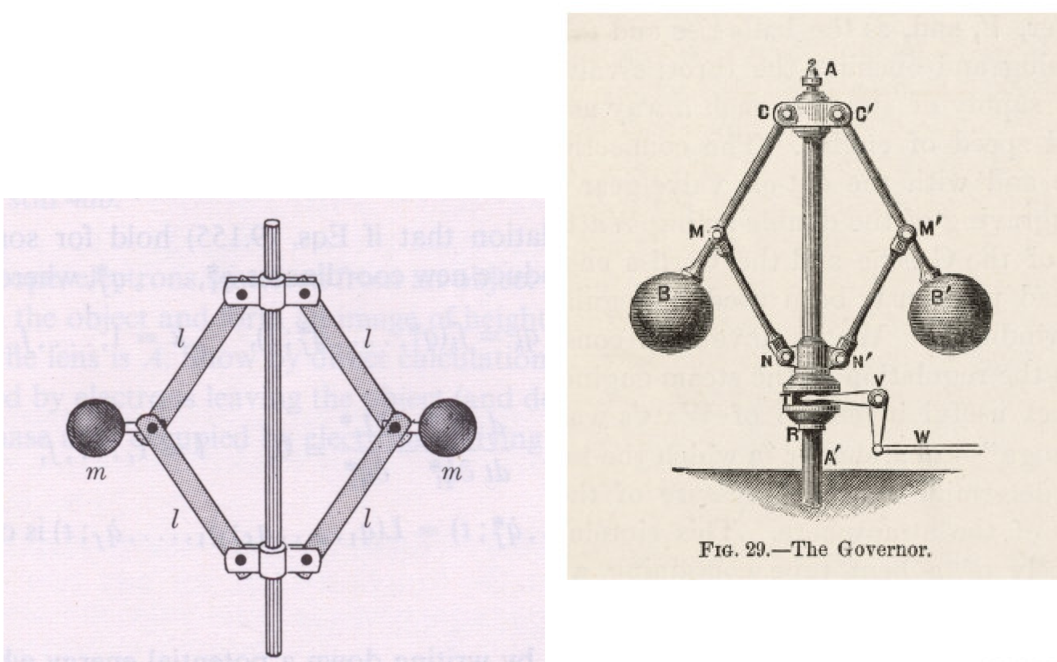

FIG. 1: The geometry of the flyball governor

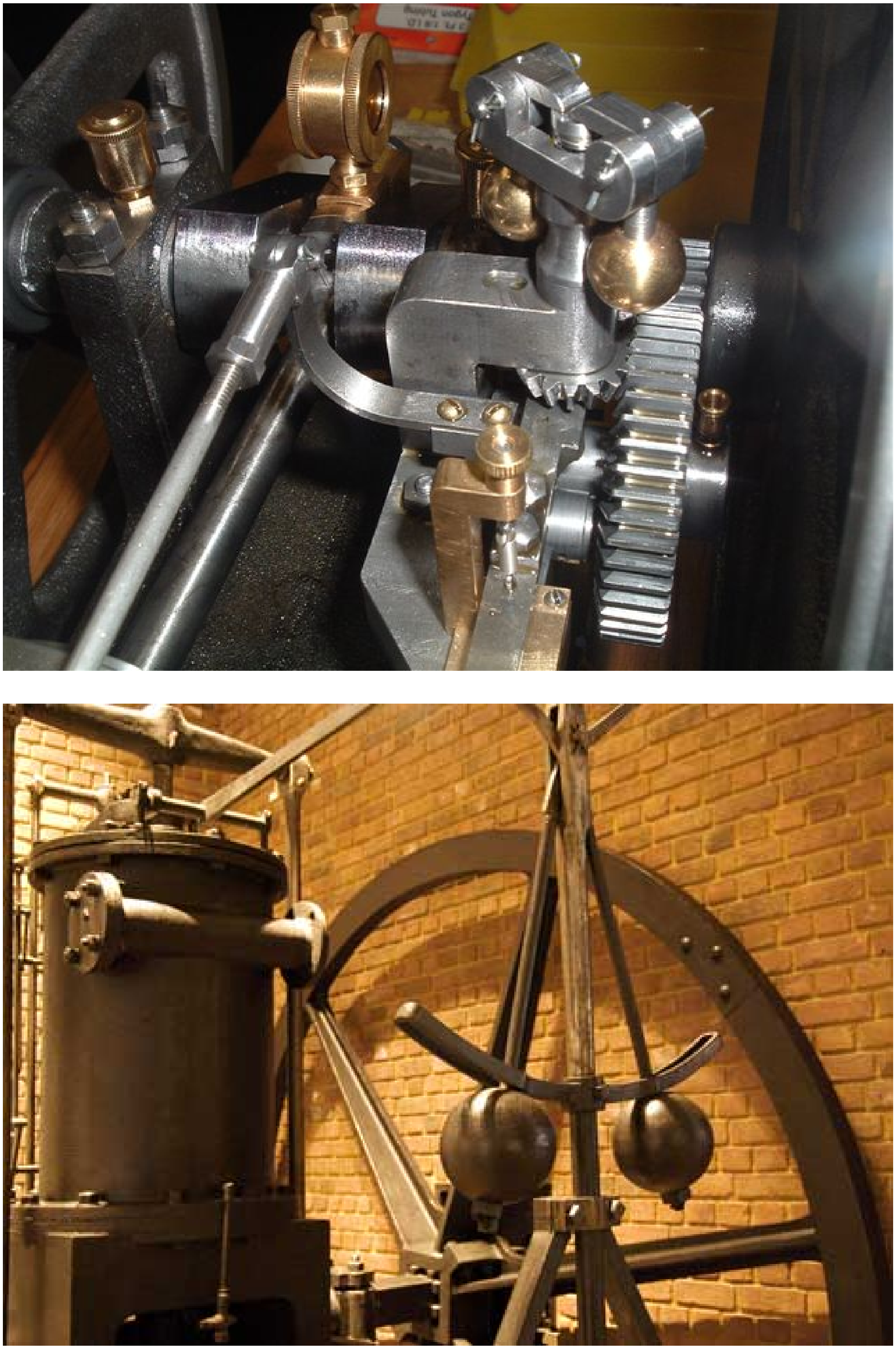

FIG. 2: Machines using the flyball governor