CSU Chico Fall Term 2018
Physics 301A

"THE TAKE AWAY" on Units and *Natural Sizes*:

1) *How to Understand It !*

- Measured numbers are comparisons between entities of like kind.
- Theories are proposed mathematical relations between these numbers.
- Since the French Revolution we have introduced standardized comparison amounts (the unit amounts) which are, however, totally arbitrary and don't relate specifically to any problem.
- With the notion of arbitrary units comes the notion of *changing* arbitrary units.
- The Victorian era Scottish physicist James Clark Maxwell introduced " unit symbols" which are actually labeled conversion factor "place-holders" for the conversion factors we will need to use when we change units. (this trick is so clever it has faked out most of the world!)
- Passively expressed equations are expressions which have not yet chosen a final set of units.
- Passive equations have "dimensional variables" and "dimensional constants" and exhibit "dimensional homogeneity".
- The mathematical structure of the variables in a physical equation carries the "Shape Knowledge" of the equation (and its scaling laws) while the constants carry the "Size Knowledge".
- **DEEP IDEA:** Every complete physical problem carries within itself a complete set of "Natural Sizes" which provide the "Natural Units" (i.e. the natural comparison amounts) for this setting. In the end, all variables will be found to be compared to these natural amounts (i.e. scaled against or "in ratio with"). (This is the Buckingham Pi Theorem …yes it always happens!)
- **DEEP IDEA:** Discerning these "Natural Units" within every physics problem and intentionally incorporating them at the very outset as we set the problem up leads to **enormously** enhanced algebraic simplicity and clarity and also leads to **absolutely essential** numeric "Well Conditioned-ness" when we use our computing machines (i.e. numbers will be "near 1").
- Once we have chosen these specific sizes as our units our equation will be "Active". An active equation has "dimensionless variables" and most constants have *disappeared* from view. An active equation is itself fully dimensionless and does not obey "dimensional homogeneity".

2) *How to Do It !*

- In standard "Passive Form" a physical number is expressed as: (pure number)·(unit symbol) which I may notate with symbols : $\#_{\mathbf{A}}\, \mathcal{U}_{\mathbf{A}}$ if I have used unit "A" .
- The conclusion of our reasoning about linear measures is that: $\#_{\mathbf{B}}\, \mathcal{U}_{\mathbf{B}} = \#_{\mathbf{A}}\, \mathcal{U}_{\mathbf{A}}$ as the central connecting relationship between numbers found from using different unit amounts.
- In Mechanics we have just three basic units: one each for length, time and mass.
- However, the constants in any given problem rarely appear as pure simple dimensions. Rather they appear most frequently as combinations: e.g. $g = 9.8\ m^1 s^{-2}$. The "accumulated unit symbols" form the "Dimensional Exponent".
- **DEEP IDEA:** We will seek super special "Natural Unit Amounts" such that the numeric values of our constants are now UNITY (the number 1 is the simplest of all numbers!).
- Call these special (and as yet unknown) "Natural Amounts" by the symbols: **L, T, M** .
- **Collect all the constants in your problem**. Now choose any three – usually you will choose key structural constants which can be constants of nature or constants of the motion or even initial conditions etc. Express them as symbols equated to exactly one unit of the required dimensional exponent in terms of **L, T, M** . Now solve *back* for **L,T,** and **M** in terms of the constants of the problem. Voila! You have done it! Use these as your units and the constants "disappear" and scale the variables and other constants in the problem.