

Reading and Problem Assignment *Revised Schedule Week 10* due Friday, April 3.

DEAR CLASS: In our new regimen you are asked to read the chapter and do these problems – but don't write them up for submission. At the end of the week I will post my own (handwritten) solutions. These are the problems you would have done in a regular semester and they exhibit the level of competency you must attain to as a technical person at this stage of your education. You will submit **only** the Portfolio Problems which are posted as a separate assignment. I intend to post the Portfolio Problems both on our class site and on Blackboard – but, as it stands now, you are to submit them on Blackboard.

I. **Potential Energy:** Please read chapter 10 in your class text.

II. ★ **Energy Estimate ... Mt. Shasta and the Sacramento River !**

Mt. Shasta is a “about 5km” high. The snow-melt from Mt. Shasta provides the icy water flow we call the Sacramento River and irrigates this bountiful valley. But something doesn't appear to add up! Consider the water that begins at the top of the mountain ... if it descended in free-fall to the valley floor ... how fast should it be moving? See if you can find some natural phenomena of comparable size. Compare that to an estimated speed of the Sacramento river. What fraction of the starting potential energy must have been lost to non-conservative work?

III. ★ **I Drink Coffee! (...and is that mechanically hazardous?)**

I'm a coffee drinker. In the morning I take a liter of water and heat it from room temperature up to boiling (i.e. from about  $20^{\circ}\text{C} \rightarrow 100^{\circ}\text{C}$ ). We are to remember that the “specific heat capacity” of water is “1” which means it takes one calorie to heat 1 gram of water by 1 degree Celcius. Since a calorie is equivalent to about 4.18 Joules, suppose on some quirky morning I decided to turn that energy which usually heats my coffee water instead into kinetic energy of the water. How fast would my coffee be traveling? Am I (and my neighbors!) still safe?

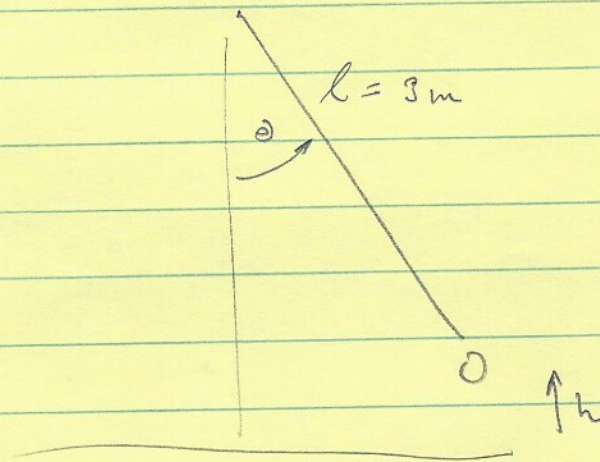
IV. ★ Problems for **Mastery**: Chapter 10 pp 255 - - **DO NOT SUBMIT !**

1. # 10, 2. # 14, 3. # 45, 4. # 52, 5. # 57, 6. # 60, 7. # 64, 8. # 69, 9. # 71

10. # 72, 11. # 73

- ✓ the single most important act in problem solving is drawing a **good picture!**
- ✓ spread out! - be neat - don't stint on space!
- ✓ **never** insert numerical values until the algebra has been worked through -relationship is *shape*.

1.) #10 pg 256



$$\text{If } \theta = 45^\circ \dots U_{\text{max}} = mgh = mgl(1 - \cos\theta)$$

$$U_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2$$

$$\text{So } mgl(1 - \cos(45^\circ)) = \frac{1}{2} m v_{\text{max}}^2$$

$$v_{\text{max}} = [2gl(1 - \cos(45^\circ))]^{1/2}$$

$$v_{\text{max}} = 4.15 \text{ m/s}$$

2.) #14 pp 256

 $1 \text{ kg}$ 

$$U = mgh = 1(9.8)(25 \text{ m}) \\ = 245 \text{ J}$$

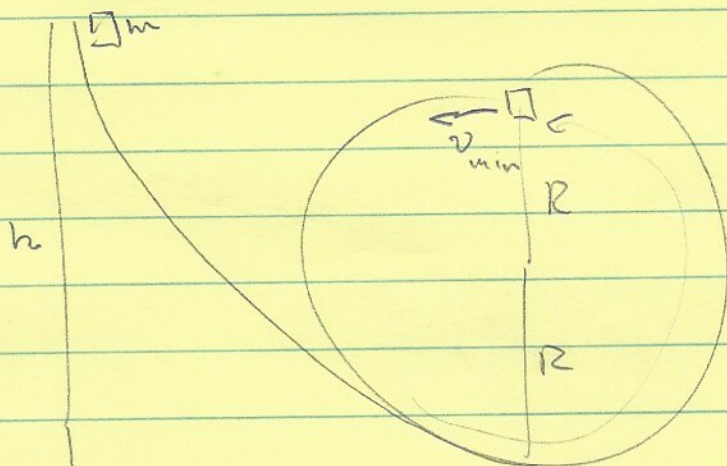
$$25 \text{ m} = h$$

$$m = 1 \text{ kg}$$

$$P = 50 \text{ MW} = 5 \times 10^7 \text{ J/sec} \quad 80\% \text{ efficient}$$

$$= \frac{5 \times 10^7 \text{ J/sec}}{(0.8) 245 \text{ J/kg}} \rightarrow 2.55 \times 10^5 \text{ kg/sec.}$$

3.) #45 pg 258



at min speed ...  
 $N = 0$  !

So  $ma = mg$

At the top  $m \frac{v_{\min}^2}{R} = mg$  so  $v_{\min}^2 = Rg$

but  $mg(h - 2R) = \frac{1}{2}mv^2$  ... so ...

$$g(h - 2R) = \frac{1}{2}v^2$$

$$\text{but } v_{\min}^2 = Rg$$

$$\Rightarrow g(h_{\min} - 2R) = \frac{1}{2}Rg$$

$$\Rightarrow \frac{h_{\min}}{R} - 2 = \frac{1}{2}$$

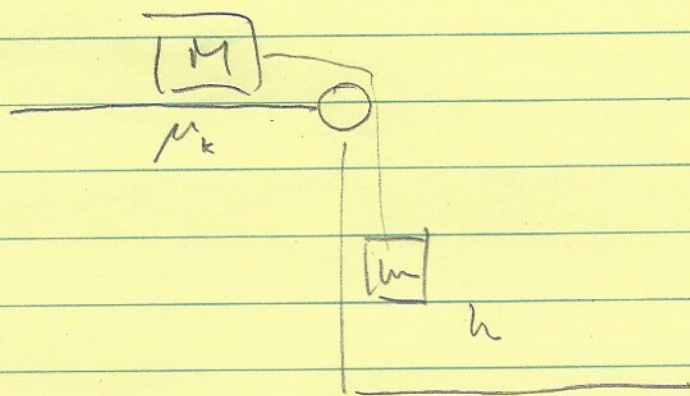
$$\frac{h_{\min}}{R} = \frac{5}{2}$$

Set 10

Knight Chap 10

4/

4.) #52 pp 258



$$\frac{1}{2}(m+M)v_f^2 = mgh - Mg\mu_k h$$

$$\text{So } v_f^2 = 2gh \left\{ \frac{m}{m+M} - \mu_k \frac{M}{m+M} \right\}$$

for a frictionless table set  $\mu_k \rightarrow \text{zero}$ .

5) # 57 pp 259

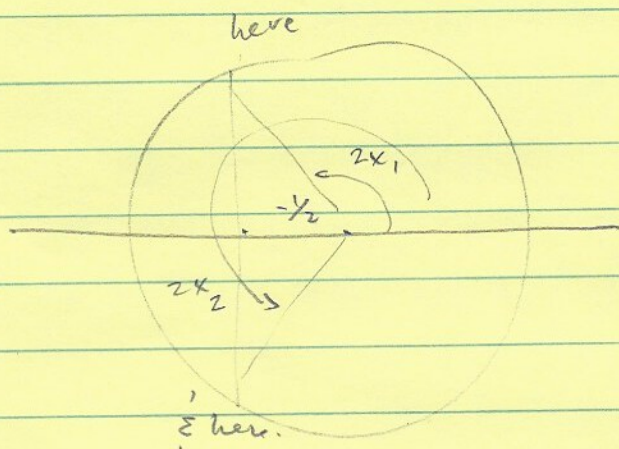
$$U(x) = \kappa + 8m \left( 2 \frac{x}{\text{meter}} \right)$$

Equilibrium means  $-\frac{\partial U}{\partial x} = 0$

Stable  $\Rightarrow \frac{\partial^2 U}{\partial x^2} > 0$

$$\frac{\partial U}{\partial x} = 1 + 2 \cos(2x) \rightsquigarrow 0 = \cos(2x) = -\frac{1}{2}$$

$$\frac{\partial^2 U}{\partial x^2} = -4 \cos(2x)$$



$$2x_1 = 120^\circ \quad x_1 = 60^\circ$$

unstable

$$2x_2 = 240^\circ \quad x_2 = 120^\circ$$

stable

$$120^\circ \longleftrightarrow \frac{2\pi}{3} \text{ rad} = 2.094 \text{ rad}$$

$$x = 2.094 \text{ m}$$

Set 10

Knight Chap 10

b) # 60 pp 259

$$F_x = -g(x-x_{eq})^3 \rightsquigarrow -gx^3$$

a)  $g \leftrightarrow \frac{N}{m^3}$

b.)  $U = -\int^x F dx = \frac{1}{4}gx^4$

$$\frac{1}{2}mv^2 = \frac{1}{4}gx^4 \rightsquigarrow \frac{1}{4}(4 \times 10^4)(.1)^4 = 1$$

so  $\frac{1}{2}(.02)v^2 = 1$

$$v^2 = 10^2 \rightsquigarrow v = 10 \text{ m/sec.}$$

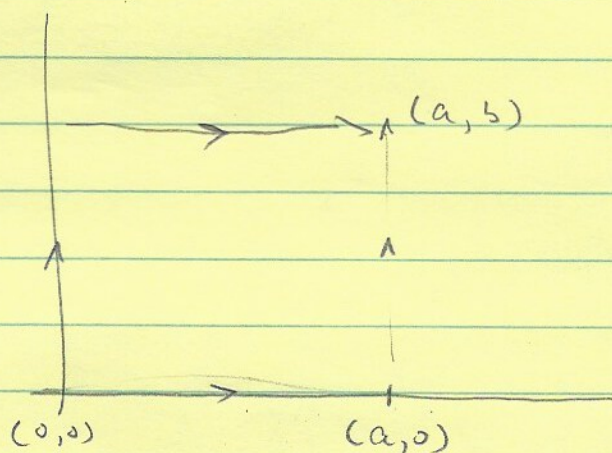
7.) # 64 pp 259

$$\vec{F} = \langle 2xy, 3y \rangle$$

$$\vec{r} = \langle x, y \rangle$$

$$d\vec{r} = \langle dx, dy \rangle$$

a)



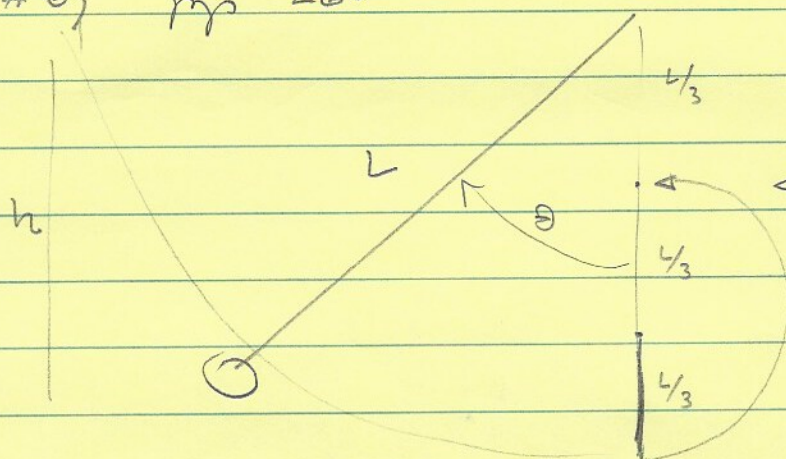
$$\begin{aligned}
 W &= \int \vec{F} \cdot d\vec{r} = \int_{(0,0)}^{(a,0)} \langle 0, 0 \rangle \cdot \langle dx, 0 \rangle = 0 \\
 &\quad + \int_{(a,0)}^{(a,b)} \langle 2ay, 3y \rangle \cdot \langle 0, dy \rangle \\
 &= \int_0^b 3y \, dy = \left. \frac{3y^2}{2} \right|_0^b = \frac{3b^2}{2}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad &\int_{(0,0)}^{(0,b)} \langle 0, 3y \rangle \cdot \langle 0, dy \rangle = \left. \frac{3y^2}{2} \right|_0^b = \frac{3b^2}{2} \\
 &+ \int_{(0,b)}^{(a,b)} \langle 2xb, 3b \rangle \cdot \langle dx, 0 \rangle = \left. \frac{2x^2b}{2} \right|_0^a = ba^2 \\
 &\text{total of } \frac{3b^2}{2} + ba^2
 \end{aligned}$$

c) Not the same  $\Rightarrow$  not conservative.



8.) #69 mp 260



← at this point we need:

$$m \frac{v^2}{L/3} = mg$$

$$\text{So! } v_{\min}^2 = \frac{L}{3} g$$

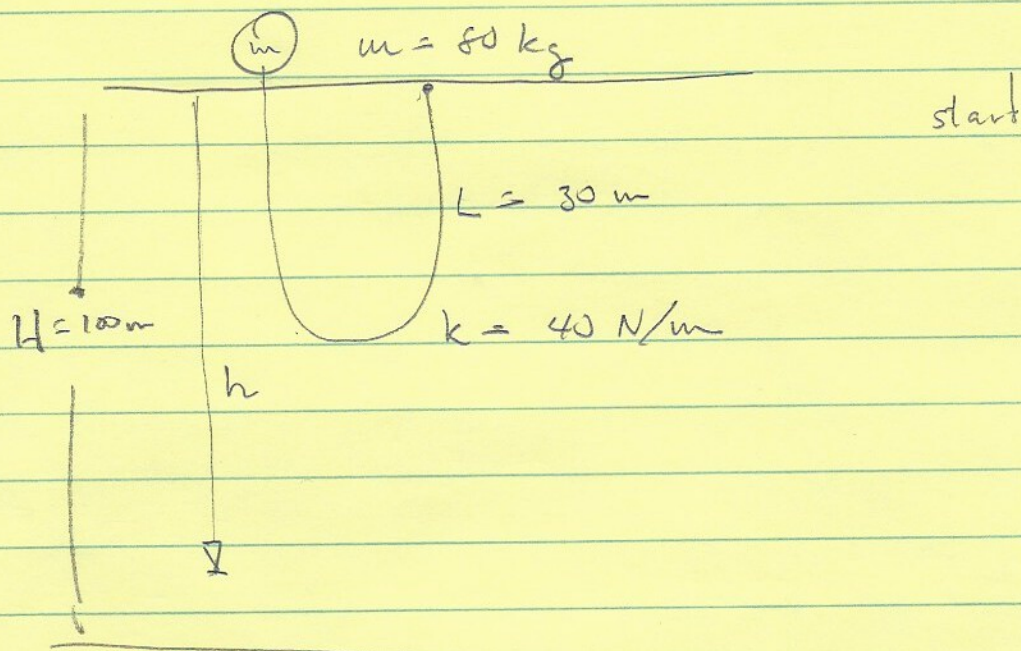
$$\text{but } mg\left(h - \frac{2L}{3}\right) = \frac{1}{2} m v^2$$

$$\text{So } g\left(h - \frac{2L}{3}\right) = \frac{1}{2} \frac{L}{3} g$$

$$h = \frac{5}{6} L = L(1 - \cos\theta)$$

$$\cos\theta = \frac{1}{6} \implies \theta = 80.4^\circ$$

9.) # 71 pp 260



Net work done = 0!

$$0 = mgh - \frac{1}{2}k(h-L)^2$$

$$\Rightarrow (h-L)^2 - \frac{2mgh}{k} = 0$$

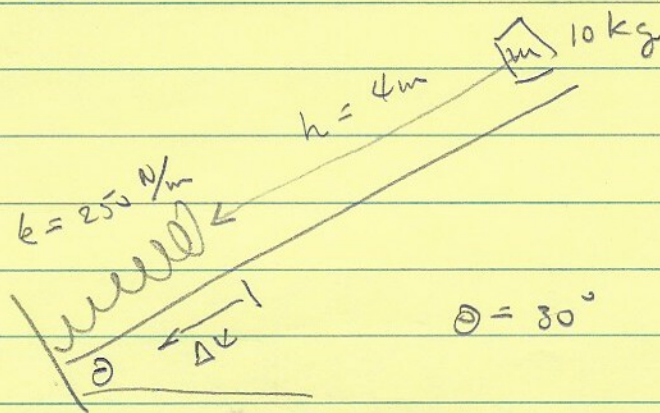
$$(h-L)^2 - 2 \frac{mg}{k}(h-L) - \frac{2mgL}{k} = 0$$

$$h-L = \frac{mg}{k} + \sqrt{\left(\frac{mg}{k}\right)^2 + 2 \frac{mgL}{k}}$$

$$h = L + \frac{mg}{k} + \sqrt{\left(\frac{mg}{k}\right)^2 + 2 \frac{mgL}{k}} = 89.1$$

Distance Over Water = 10.9 m

10.) # 72 pg 260



$$\text{net work} = \text{zero} ; \quad m g \sin \theta (h + \Delta x) = \frac{1}{2} k \Delta x^2$$

$$\Delta x^2 - \frac{2 m g \sin \theta}{k} (\Delta x + h) = 0$$

$$\Delta x^2 - \left(\frac{98}{250}\right) \Delta x - \left(\frac{98}{250} \times 4\right) = 0$$

.392

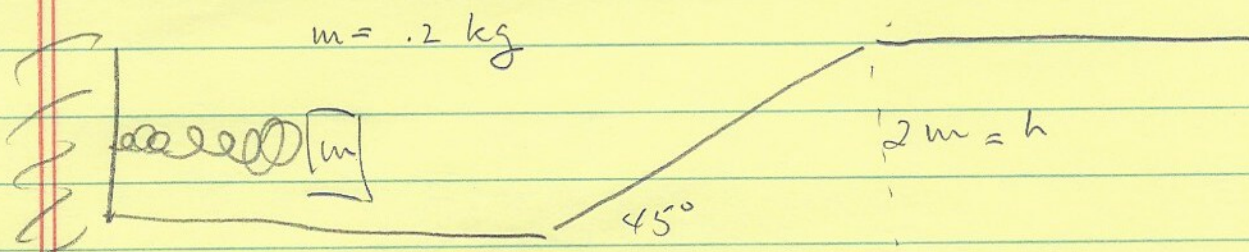
$$a) \Delta x_{\text{max}} = .392 \pm \sqrt{(.392)^2 + 6.272} = 2.93 \text{ m}$$

b)  $v_{\text{max}}$  when  $a = 0$ 

$$k \Delta x = m g \sin \theta \quad \text{or} \quad \Delta x = \frac{m g}{k} \left(\frac{1}{2}\right)$$

$$\Delta x = \frac{10 (9.8)}{250} \frac{1}{2} = \frac{9.8}{50} = .196 \text{ m}$$

11.) #73 pr 260



$$E_0 = \frac{1}{2} k \Delta x^2 = \frac{1}{2} (10^3) (.15 \text{ m})^2 = 11.25 \text{ J}$$

if all becomes kinetic

moving up the slope we lose KE!

$$F_k = \mu_k mg \cos \alpha = (.2) (.2) (9.8) \frac{1}{\sqrt{2}} = .392 \frac{1}{\sqrt{2}}$$

$$\Delta W_{\text{fric.}} = F_k (2\sqrt{2} \text{ m}) = \frac{.392}{\sqrt{2}} 2\sqrt{2} = .784 \text{ J}$$

$$\Delta W_{\text{grav}} = mg(h) = (.2) (9.8) 2 = 3.92 \text{ J}$$

So! at the hill top  $E_f = 11.25 - .784 - 3.92 = 6.546 \text{ J}$

$$\text{range?} = \frac{v_0^2}{g} \quad \text{but} \quad \frac{1}{2} m v_0^2 = E_f$$

$$\text{So} \quad \frac{2E_f}{mg} = \frac{v_0^2}{g} = \text{range}$$

$$= \frac{6.546}{(.2)(9.8)} = 3.34 \text{ m}$$