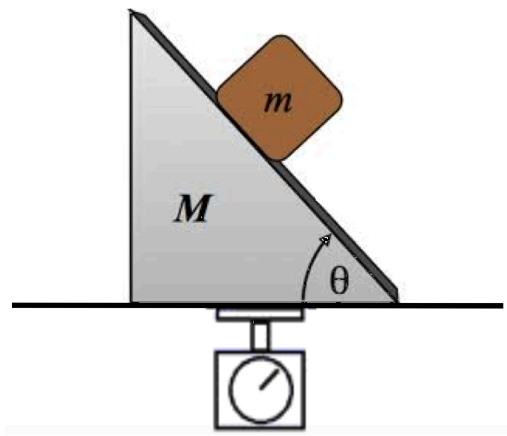
CSUC Spring Term 2020 Physics 204A **Portfolio Problem for Week 10:**

Solutions

The scenario we start with is a block of mass $\boldsymbol{m}$ sitting on a wedge of mass $\boldsymbol{M}$ and the whole is on a board which sits on a scale attached to the floor. We begin very simply and then start removing constraints … each time asking what the outcome will be.



1) If the slanted surface of the wedge is frictionless but the wedge itself is firmly attached to the board … and we release the block, we now ask:

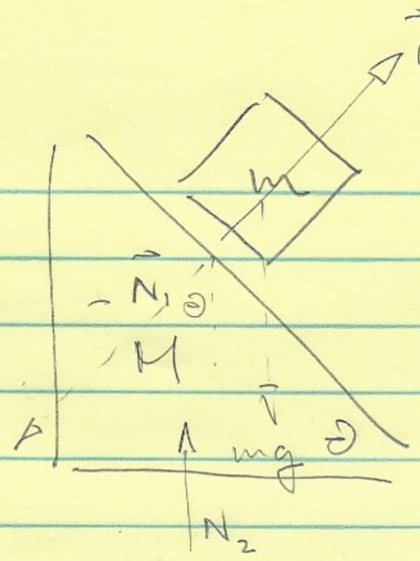
1) What is the acceleration of the block down the wedge ?
2) What does the scale read as the block slides down the wedge?

2) Suppose now that the slanted surface of the wedge has a kinetic frictional coefficient μ_k with the block. If we release the block, it still slides down the wedge, but we still ask:

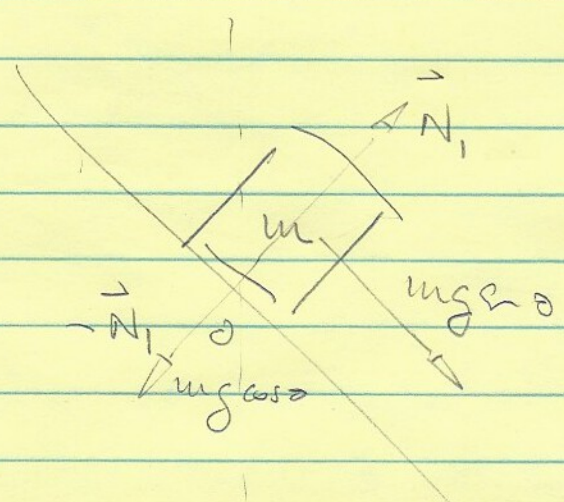
1) What is the acceleration of the block down the wedge now ?
2) What does the scale read as the block slides down the wedge now?

3) Suppose that we now remove the friction on the slope but then we also remove the friction between the wedge and the board. If we release the block, it still slides down the wedge only now … the wedge moves to the *left* on the board too! We examined something like this in class.

1) What is the acceleration of the block now ?
2) What is the acceleration of the wedge ?
3) What does the scale read as the block slides down the wedge now?



1)



$ma = mg \sin\theta$

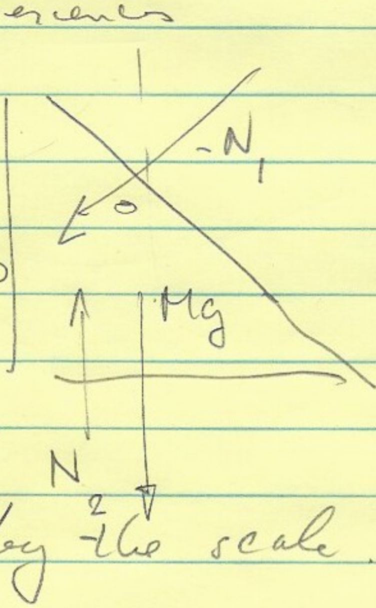
so $\frac{a}{g} = \sin\theta$

The wedge exerts $|\vec{N}_1| =$ mg cos θ onto the mass
the mass exert $-\vec{N}_1$ back

the wedge experiences

? y) $N_2 - Mg - N_1 \cos\theta$

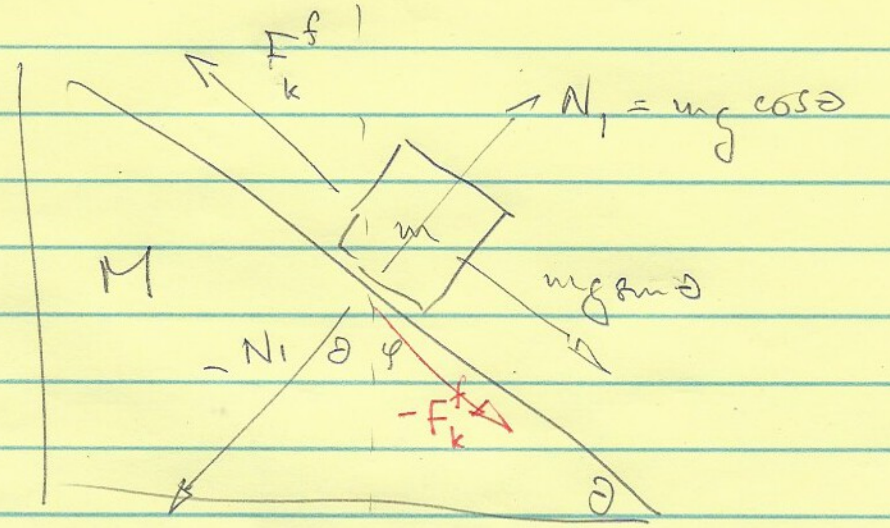
$= N_2 - Mg - mg\cos^2\theta = 0$



So! $N_2 = Mg + mg\cos^2\theta$

This is delivered by the scale!

2)



$$F_k^f = \mu_k N_1 = \mu_k mg\cos\theta$$

The downward force components on the wedge:

$$= -Mg - N_1\cos\theta - F_k^f\cos\varphi$$

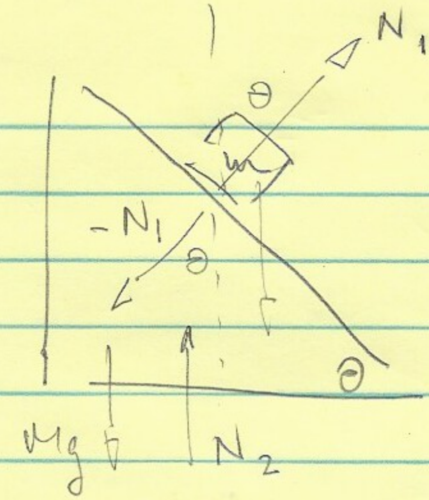
$$= -\left(Mg + mg\cos^2\theta + \mu_k mg\cos\theta\cos\varphi \right)$$

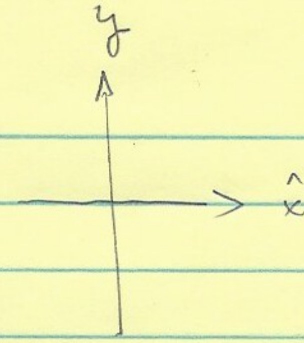
$$= -\left(M + m\cos^2\theta + \mu_k m\cos\theta\sin\theta \right) g$$

This must be the upward force from the scale!

The block obeys $ma = mg\sin\theta - \mu_k mg\cos\theta$

$$\frac{a}{g} = \left(\sin\theta - \mu_k\cos\theta \right)$$





block $\hat{x}$) $m a_x = N_1 \sin\theta$

$\hat{y}$) $m a_y = -mg + N_1 \cos\theta$

Wedge $\hat{x}$) $M A_x = -N_1 \sin\theta$

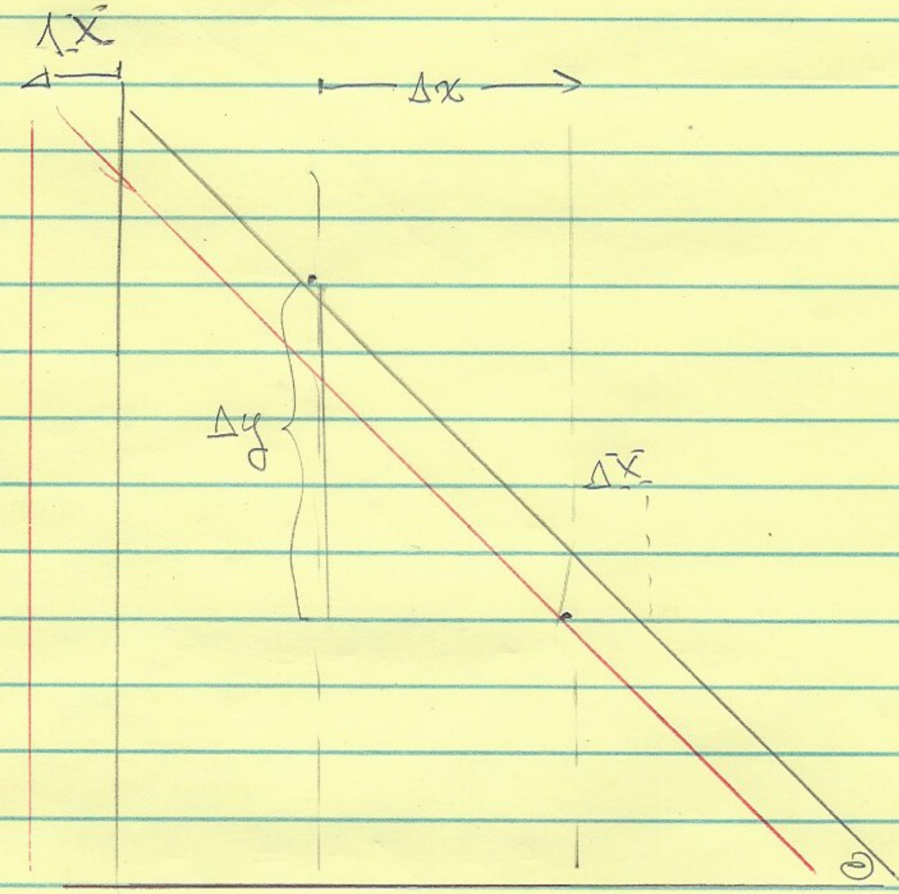
$\hat{y}$) $0 = M A_y = -Mg - N_1 \cos\theta + N_2$

Since $M A_x = -m a_x \ldots$

$M V_x = -m v_x$

$M \Delta X = -m \Delta x$

but



$$\frac{\Delta y}{\Delta x - \Delta \bar{X}} = \tan\theta$$

and $\Delta \bar{X} = -\frac{m}{M}\Delta x$

so $$\frac{\Delta y}{\Delta x\left(1+\frac{m}{M}\right)} = \tan\theta$$

$$\frac{\Delta y}{\Delta x} = \left(1+\frac{m}{M}\right)\tan\theta = \frac{v_y}{v_x} = \frac{a_y}{a_x}$$

but $\dfrac{a_y}{a_x} = \dfrac{-mg + N_1 \cos\theta}{N_1 \sin\theta}$

So! $\left(1+\dfrac{m}{M}\right)\tan\theta = \dfrac{-mg + N_1\cos\theta}{N_1 \sin\theta}$

$$\Rightarrow N_1 \frac{\sin^2}{\cos}\left(1+\frac{m}{M}\right) = -mg + N_1\cos\theta$$

$$mg\cos\theta = N_1\left\{\sin^2\left(1+\frac{m}{M}\right) + \cos^2\right\}$$

$$mg\cos\theta = N_1\left\{1 + \frac{m}{M}\sin^2\theta\right\}$$

$$\frac{mg\cos^2\theta}{1+\frac{m}{M}\sin^2\theta} = N_1\cos\theta$$

The Scale weighs $\underline{N_2 = Mg + N_1\cos\theta}$

The scale weighs $\underline{Mg + mg\left(\dfrac{\cos^2\theta}{1+\frac{m}{M}\sin^2\theta}\right)}$

$$\vec{a} = \langle a_x , a_y \rangle$$

$$= g \left\langle \frac{\cos\theta \sin\theta}{1 + \frac{m}{M}\sin^2\theta} , -1 + \frac{\cos^2\theta}{1 + \frac{m}{M}\sin^2\theta} \right\rangle$$

if $M \to \infty$?

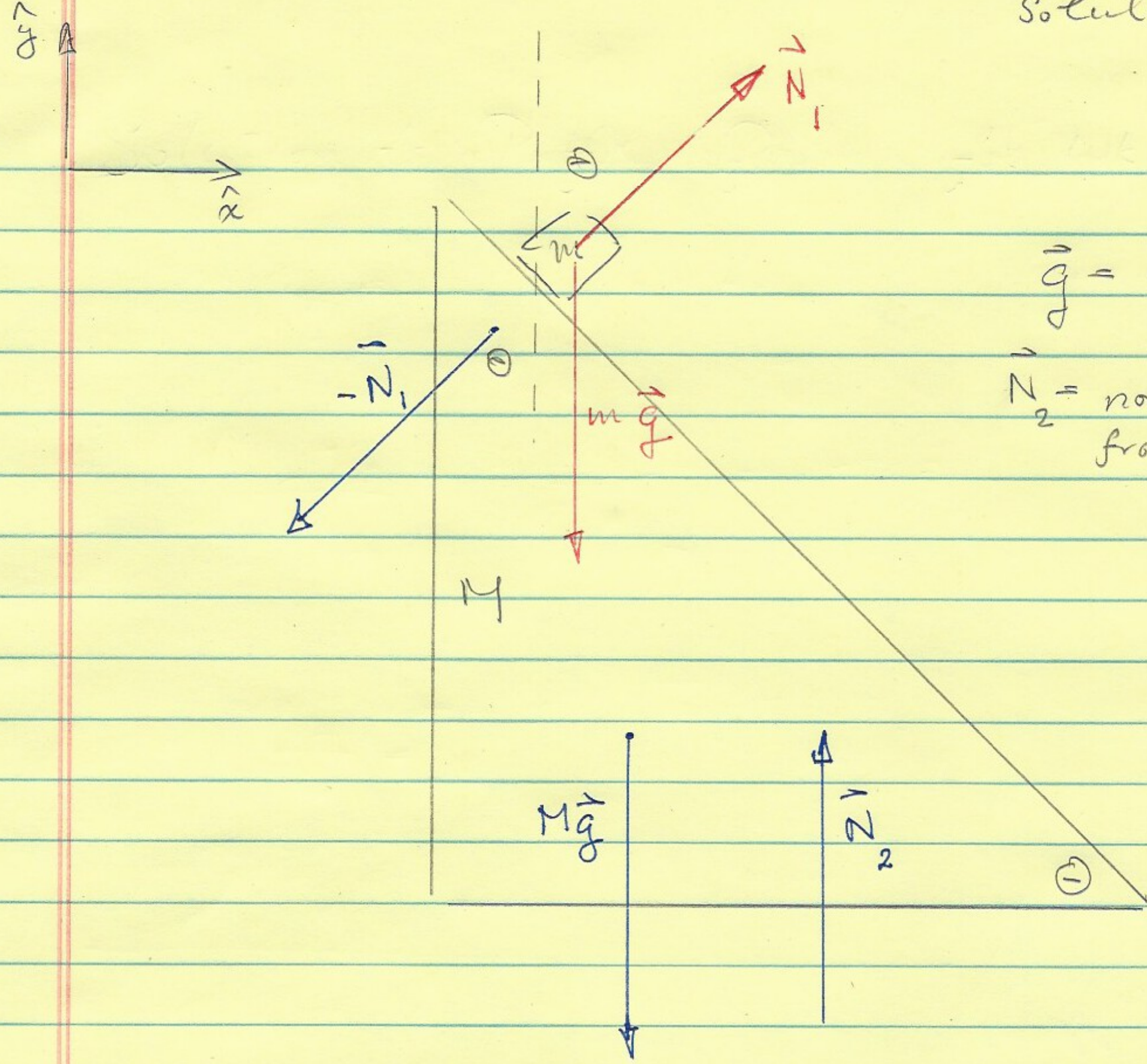
$$= g \langle \cos \sin , -\sin^2 \rangle$$

$$= g \sin\theta \langle \cos\theta , -\sin\theta \rangle \quad \text{correct !}$$

$$\frac{g}{1 + \frac{m}{M}\sin^2\theta} \left\langle \cos\theta \sin\theta , -\left(1 + \frac{m}{M}\sin^2\theta\right) + \cos^2\theta \right\rangle$$

$$'' \quad \left\langle \cos\theta \sin , -\sin^2\theta \left(1 + \frac{m}{M}\right) \right\rangle$$

$$\vec{a} = \frac{g \sin\theta}{1 + \frac{m}{M}\sin^2\theta} \left\langle \cos\theta , -\sin\theta\left(1 + \frac{m}{M}\right) \right\rangle$$



$\vec{g} = -g\hat{y}$

$\vec{N}_2$ = normal force from scale!

Forces in our System:

m) m experiences gravity and the normal force $\vec{N}_1$

M) M experiences gravity & $-\vec{N}_1$ (3rd law!) as well as the normal force from the scale $\vec{N}_2$

(red ⟷ forces on m
blue ⟷ forces on M)

Now, apply Newton's 2^{nd} Law: write $m\vec{a} = \vec{F}^{net}$ for both masses.

I call $N_1 \equiv |\vec{N}_1|$ positive magnitudes
$N_2 \equiv |\vec{N}_2|$

$\hat{x}$)

$$\begin{cases} 1)\quad m a_x = N_1 \sin\theta \\ 2)\quad M A_x = -N_1 \sin\theta \end{cases}$$

$\hat{y}$)

$$\begin{cases} 3)\quad m a_y = -mg + N_1 \cos\theta \\ 4)\quad M A_y = 0 = N_2 - Mg - N_1 \cos\theta \end{cases}$$

✓ Notice $N_2 = Mg + N_1 \sin\theta$ ← "scale reading"

✓ # of unknowns? a_x, A_x, a_y, N_1, N_2

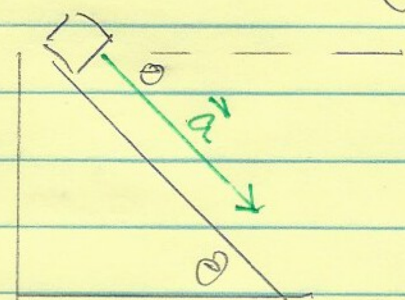
?? 5 unknowns ... but only 4 equations!

BAD!

but wait? How did we solve this before?

Previously ... we assumed the wedge didn't move!

This meant mass m made its way down the slope at angle θ

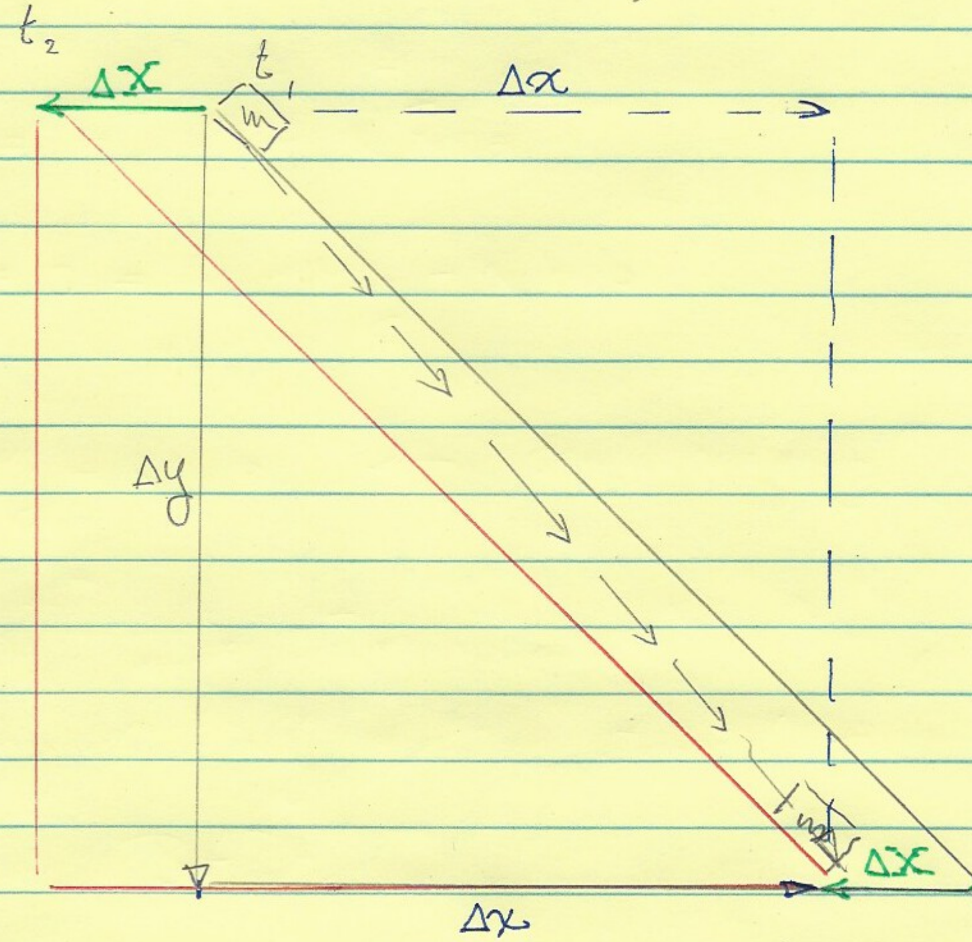


$a_x = |\vec{a}| \cos\theta$

$a_y = -|\vec{a}| \sin\theta$

so ... $\frac{a_y}{a_x} = -\tan\theta$ ... this was our 5^{th} equation.

... but now ...
Because the wedge scoots "left" ... the mass doesn't travel down the slope at angle θ!



In fact ... now ...

$$\frac{\Delta y}{\Delta x - \Delta X} = -\tan\theta$$

I've drawn the wedge at two times: when the mass is at the top ... and when it's at the bottom.

Previously $\dfrac{\Delta y}{\Delta x} = -\tan\theta$

Now $\dfrac{\Delta y}{\Delta x - \Delta X} = -\tan\theta$

but wait... we know $m a_x = -M A_x$ (3^{rd} law)

so ... $m v_x = -M V_x$ ... so ...

$$m \Delta x = -M \Delta X$$

$\Longrightarrow \Delta X = -\dfrac{m}{M} \Delta x$!

So $\dfrac{\Delta y}{\Delta x (1 + \frac{m}{M})} = -\tan\theta$

$\Longrightarrow \Delta y = -(1 + \frac{m}{M}) \tan(\theta)\, \Delta x$ (take derivatives)

$\Longrightarrow a_y = -(1 + \frac{m}{M}) \tan(\theta)\, a_x$

This is our 5^{th} equation!

It's modified ... since $M < \infty$ and moves now.
$\Longrightarrow$ we "see it".

Solve! (the five equations). [just "DO IT"]

I get:

$$\frac{a_x}{g} = \frac{\sin\theta\cos\theta}{1+\frac{m}{M}\sin^2\theta}$$

$$\frac{N_1}{mg} = \frac{\cos\theta}{1+\frac{m}{M}\sin^2\theta}$$

$$a_y = -\left(1+\frac{m}{M}\right)\tan\theta \; a_x$$

$$A_x = -\frac{m}{M}a_x$$

$$N_2 = Mg + N_1\sin\theta$$

Scale: $Mg + mg\dfrac{\cos^2\theta}{1+\frac{m}{M}\sin^2\theta}$

Check!

If $\frac{m}{M} \to$ "small" ... we had better reduce to the solution we had in the 1st part (M doesn't move!)

If $m/M \to$ "zero" then

$a_x \to g \sin\theta\,(\cos\theta)$

$a_y \to g \sin\theta\,(-\sin\theta)$

so $\frac{a_y}{a_x} = -\tan\theta$

$|\vec{a}| = g\sin\theta$

$N_1 \to mg\cos\theta$

$A_x \to$ zero

$N_2 \to Mg + mg\cos^2\theta$

Yes! These all become the previous solutions

It works.