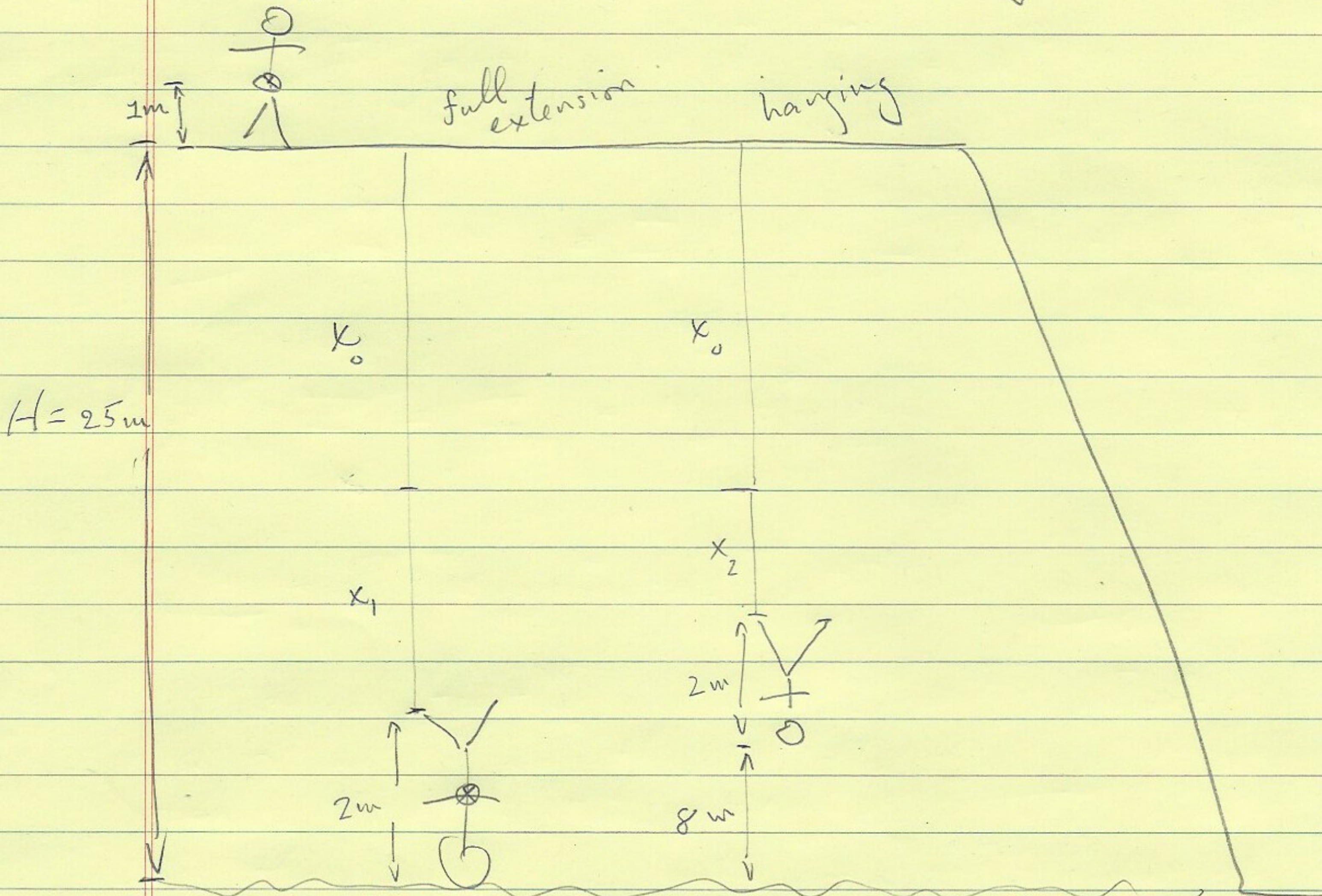
Portfolio Problem for Week 11: Solution.

✓ We have a person of height 2m whose Center of Mass is midway up (1m). The mass M of this person is not known.

✓ We need a bungee cord of, as yet, unknown rest length x_0 and unknown spring constant k.



x_0 = rest length

x_1 = max stretch

x_2 = rest stretch

There are two kinds of potential energy here:

$$\Delta U_{grav} = Mg\,\Delta y$$

$$\Delta U_{spring} = \tfrac{1}{2} k\, \Delta x^2 \qquad \text{for stretch } \Delta x.$$

1.) This is a <u>conservative</u> system: $U_{tot} = U_{grav} + U_{spring}$

We watch the jump! $E_f = E_i$

$$\Rightarrow (U_{tot} + KE)_f = (U_{tot} + KE)_i$$

but $KE_f = KE_i = 0 \rightsquigarrow U_{tot_f} = U_{tot_i}$

So $\Delta U_{grav} + \Delta U_{spring} = 0$

$$(-25\,m)\,Mg + \tfrac{1}{2} k\, x_1^2 = 0$$

2.) <u>While hanging</u> $F_{net} = 0$, so ...

$$k|x_2| - Mg = 0$$

3) $x_0 + |x_2| = 15\ m$

4) $x_0 + |x_1| = 23\ m$

$\Big\}$ let x_0, x_1, x_2 be positive numbers.

1) $x_1^2 = 50m\ Mg/k$

2) $x_2 = Mg/k$

3) $x_0 + x_2 = 15m$

4) $x_0 + x_1 = 23m$

4) − 3) so! $x_1 - x_2 = 8m$ or $x_1 - 8m = x_2$

1) & 2) $x_1^2 = (50m)\ x_2$

so $x_1^2 = (50m)\ (x_1 - 8)$

$x_1^2 - 50\ x_1 + 400 = 0$

$x_1 = \frac{50 \pm \sqrt{2500 - 1600}}{2} = \frac{50 \pm \sqrt{900}}{2}$

so $x_1 = \frac{50 \pm 30}{2}$ so 40 or 10

$x_2 = 32$ or 2

$x_0 = 23 - x_1$ so $x_0 = 23 - 10 = \underline{13}$

✓ $x_1 = 10m$

✓ $x_2 = 2m$

✓ $Mg/k = 2m$ $\frac{k}{M} = \frac{g}{2m}$

Since $Ma = F^{net}$ ... the maximum speed must occur when $F^{net} = 0$ since if F^{net} were positive we'd still be speeding up and if it were negative we'd be slowing down.

$$F^{net} = -Mg + k|\Delta x| \rightsquigarrow \text{zero!}$$

but $F^{net} = 0$ is also the hanging pt.

$$\Delta x_{rest} = x_2 = 2\,m$$

Since $KE_f - \cancel{KE_i}^{\,0}$ = Net work done

$$\tfrac{1}{2} M v_f^2 = +Mg\,(17m) - \tfrac{1}{2}k\,(2m)^2$$

$$v_f^2 = 2g\,(17m) - \frac{k}{M}\,(2m)^2$$

but $\dfrac{k}{M} = \dfrac{g}{2m}$

$$v_f^2 = 2g\,(17m) - g\,(2m)$$

$$v_f^2 = 32\,g \cdot m$$

$$v_f = \sqrt{32\,gm} = 17.7\ m/s$$

Happens at bottom of P.E. curve! $U + KE = \text{const}$

$U \rightarrow \min \Rightarrow KE \rightarrow \max$.

Max acceleration ?

$Ma = F^{net} \Rightarrow a^{max} \longleftrightarrow (F^{net})^{max}$.

$a = -g + \frac{k}{M}\Delta x \qquad \Delta x = $ "stretch".

$\frac{K}{M} = \frac{g}{2m}$

$a^{max} = -g + \frac{g}{2m}\Delta x^{max}$ but $\Delta x^{max} = x_1 = 10m$

$a^{max} = -g + 5g = 4g$ upwards.

It happens at the bottom.

rest length $x_0 = 13$ m

x_1 max stretch $= 10$ m

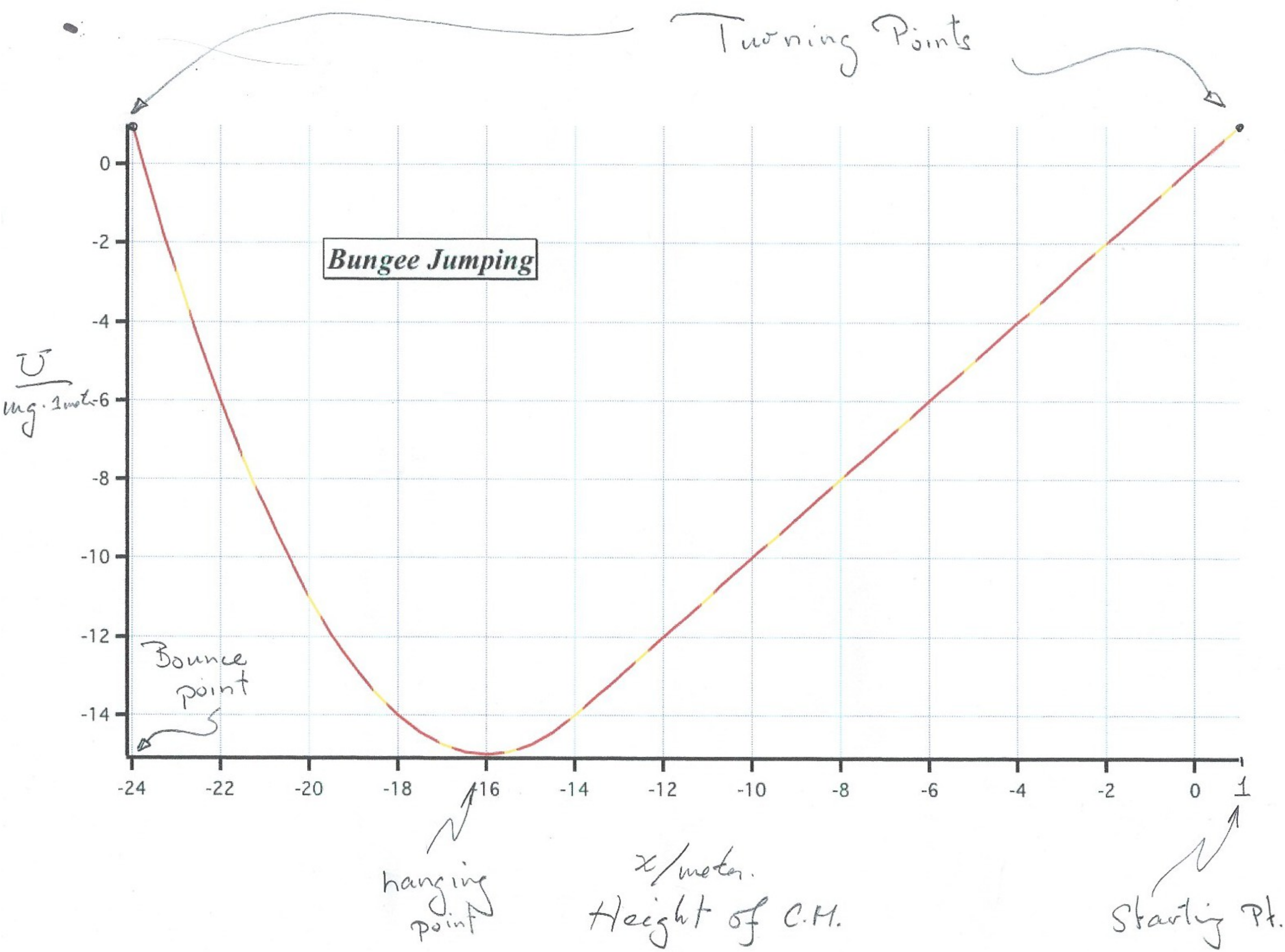
x_2 stretch at rest $= 2m$

$\frac{K}{M} = g/_{2\text{meter}}$

$v^{max} = 17.7$ m/s at 2 meter stretch

$a^{max} = 4g$ upwards at max stretch $= 10m$.

Not enough to kill you ... just enough to thrill you!



Turning Points
Bungee Jumping
U / mg · 1 meter
0
-2
-4
-6
-8
-10
-12
-14
Bounce point
-24
-22
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
1
hanging point
x/meter.
Height of C.M.
Starting Pt.