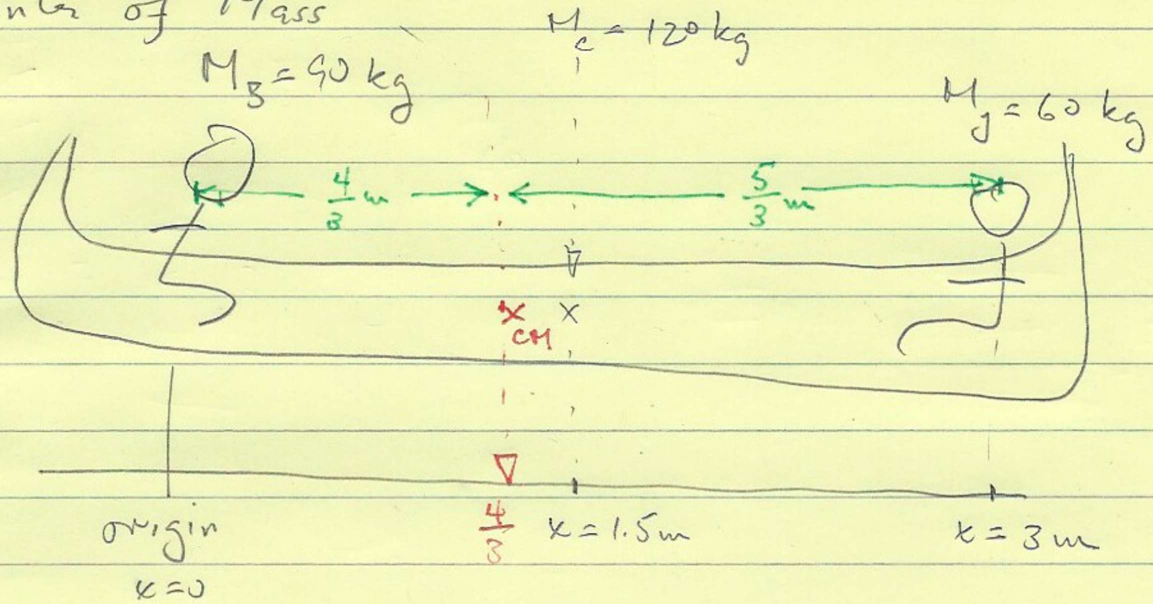
Center of Mass



$$M_{tot}\, X_{com} = \sum_i m_i x_i = 90(0) + 120(1.5) + 60(3)$$

$$270\, X_{com} = 0 + 180 + 180$$

a) $$X_{com} = \frac{360}{270} = \frac{4}{3} \text{ meter}$$

2nd solution: $M_B = M_J + \Delta M = 60 + 30$

It is "as if" we had

$$\underset{+\Delta M}{M_J} \;—\; M_c \;—\; M_J$$

$$= \underset{0}{\Delta M} \longrightarrow \underset{1.5\,m}{(2M_J + M_c)}$$

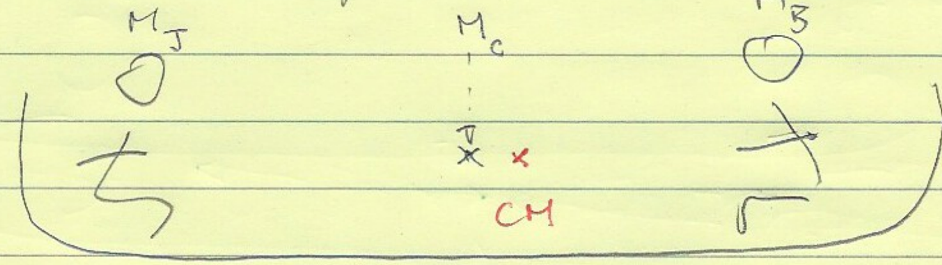
a') So $270\, X_{com} = (30)(0) + 240(1.5) = 360$

$$X_{com} = \frac{360}{270} = 4/3 \quad \text{same.}$$

Center of Mass (cont.)

Now Switch positions!

b)



The Center of mass has <u>NOT</u> moved.

The distance from Bill to CM is $\frac{4}{3}$ meter

The distance from Jill to CM is $\frac{5}{3}$ meter

c) Bill <u>was</u> $\frac{4}{3}$ m to the <u>left</u> of CM and is now $\frac{4}{3}$ m right

Jill <u>was</u> $\frac{5}{3}$ m right of CM... and is now $\frac{5}{3}$ m left.

The center of the canoe <u>was</u> $\frac{1}{6}$ m to the right of CM and is now $\frac{1}{6}$ m left of CM.

So! The canoe moved $\frac{2}{6} = \frac{1}{3}$ m left!

Bill moved $\frac{8}{3}$ m right

Jill moved $\frac{10}{3}$ m left

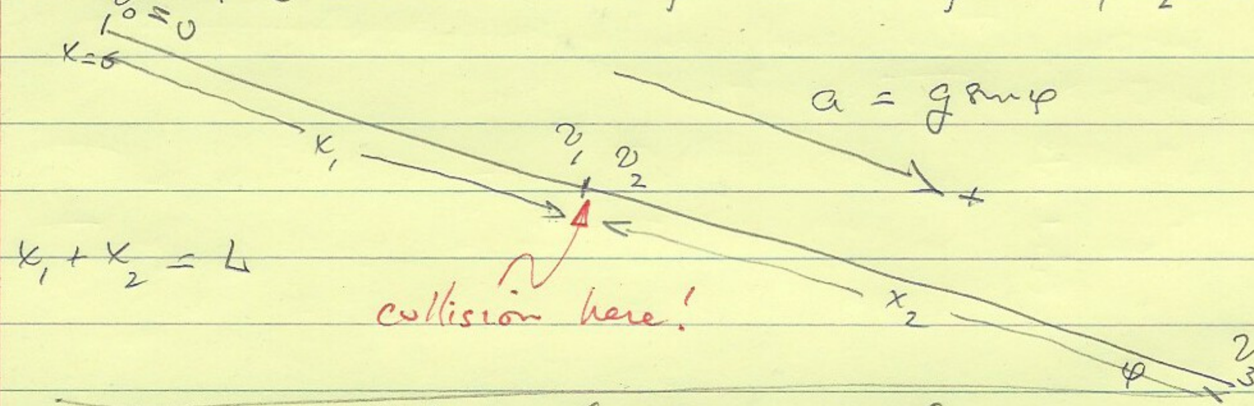
$V_{Bill} = \frac{8}{9}$ m/s "right" $V_J = \frac{10}{9}$ m/s "left" $V_{canoe} = \frac{1}{9}$ m/s "left"

2) Inelastic Collisions

In any inelastic collision

$$(m_1) \xrightarrow{v_i} (m_2) \longrightarrow (m_1 \, \} \, m_2) \xrightarrow{v_f}$$

$$m_1 v_i = (m_1 + m_2)\, v_f \quad \text{so} \quad v_f = \frac{m_1}{m_1+m_2} v_i$$



In any constant acceleration situation

$$v_f^2 - v_i^2 = 2a\Delta x$$

We know: ✓ $v_1^2 - \cancel{v_0^2}^{\,0} = 2ax_1$

✓ $v_3^2 - v_2^2 = 2ax_2$ } add

$$v_3^2 - v_2^2 + v_1^2 = 2aL \quad \text{and} \quad v_2 = \left(\frac{m_1}{m_1+m_2}\right) v_1$$

$$v_3^2 = 2aL - (v_1^2 - v_2^2) = 2aL - v_1^2\left(1 - \left(\frac{m_1}{m_1+m_2}\right)^2\right)$$

$$\text{So} \quad v_3^2 = 2aL - 2ax_1\underbrace{\left[1 - \left(\frac{m_1}{m_1+m_2}\right)^2\right]}_{\text{positive } \# < 1}$$

So we conclude

$$v_3^2 = 2aL\left\{ 1 - \frac{x_1}{L}\left[1 - \left(\frac{m_1}{m_1+m_2}\right)^2\right] \right\}$$

In numbers this is

$$2gL\sin(20°)\left\{1 - \frac{1}{2}\left(1 - \left(\frac{90}{150}\right)^2\right)\right\}$$

$$2gL\sin(20°)\left\{1 - \frac{1}{2}\left(1 - \left(\frac{3}{5}\right)^2\right)\right\}$$

$$2gL\sin(20°)\left\{.68\right\} = 911.7 \quad \text{so } v_3 = 30.19 \text{ m/s}$$

5) We make v_3 smallest by making $\frac{x_1}{L}$ greatest $= 1$

$$v_{3_{min}}^2 = 2gL\sin(20°)\left\{1 - \left[\left(1 - \left(\frac{3}{5}\right)^2\right)\right]\right\}$$

$$2gL\sin(20°)(.36) = 482.66$$

$v_{3_{min}} = 21.97$ m/s about 27% smaller.

Quite a Show!