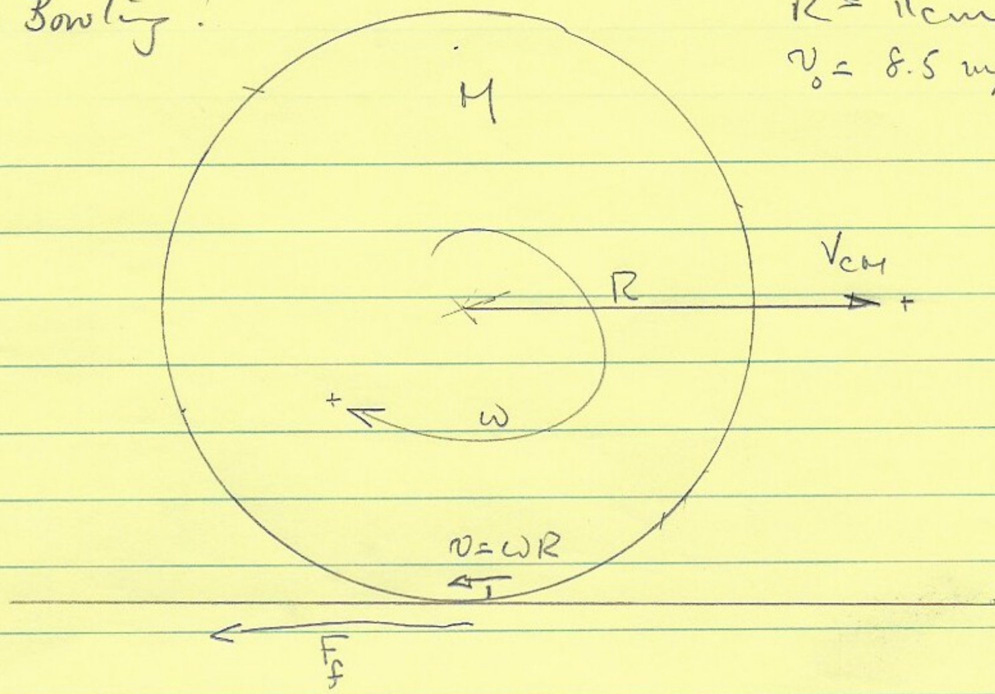
Let's go Bowling!

$R = 11\text{cm}$
$v_0 = 8.5\ m/s$

I.



Eq. of Motion

Translation: $$M \frac{dV_{CM}}{dt} = -F_f$$

Rotation: $$I \frac{d\omega}{dt} = F_f R$$

The C.M. has $V_{CM} = v_0 - \frac{F_f}{M} t$

Rotation has $\omega = \frac{F_f R}{I} t$

The point of contact is moving at velocity
$$v_{contact} = V_{CM} - \omega R$$

When the contact point is not skidding ... $v_{contact} \rightarrow 0$.
and friction stops!

This happens when $V_{CM} = \omega R$ or ...
$$v_0 - \frac{F_f}{M} t_c = \frac{F_f R^2}{I} t_c$$

I. cont.

Now $I_{cm} = MR^2 \#$ (# = "magic number)

So $v_0 = \frac{F_f}{M}\left(1 + \frac{1}{\#}\right) t_c$

a) So $v_0 \frac{M}{F_f}\left(\frac{\#}{1+\#}\right) = t_c$ is how long skidding happens.

c) Final velocity? $v_f = V_0 + a t_c = V_0 - \frac{F_f}{M} t_f$

$$v_f = v_0\left[1 - \frac{\#}{1+\#}\right] = v_0\left(\frac{1}{1+\#}\right)$$

b) Distance? $v_f^2 - v_0^2 = 2 a \Delta x$

So ... $v_0^2\left(1 - \left(\frac{1}{1+\#}\right)^2\right) = 2 \frac{F_f}{M} \Delta x$

$$\Longrightarrow \Delta x_{skid} = \frac{\frac{1}{2} M v_0^2}{F_f}\left(1 - \left(\frac{1}{1+\#}\right)^2\right)$$

Since $F_f = \mu_k M g$

a) $v_0 \frac{1}{\mu_k g}\left(\frac{\#}{1+\#}\right) = t_c = \frac{v_0}{\mu_k g} \frac{2}{7}$

b) $\Delta x_{skid} = \frac{1}{2} \frac{v_0^2}{\mu_k g}\left(1 - \frac{1}{(1+\#)^2}\right) = \frac{12}{49} \frac{v_0^2}{\mu_k g}$

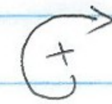
c) $v_f = v_0\left(\frac{1}{1+\#}\right) = \frac{5}{7} v_0$

Rolling

Universal Equations

1) $M a_{CM} = F^{net}$

2) $I_{CM} \alpha_{CM} = \tau_{CM}$



3) Parallel axis thm.

$\longrightarrow +$

a) use 1) $(.2\,kg)(8\,m/s^2) = 1.4\,N + F_f$

so! $1.6\,N = 1.4\,N + F_f \rightsquigarrow F_f = .2\,N$ (to the right!)

b) $I_{CM} \alpha_{CM} = F^A R_1 - F_f R_2$ but $R_2 \alpha = a_{CM}$ (mult. by R_2)

$$I_{CM} a_{CM} = R_2 \left(F^A R_1 - F_f R_2 \right)$$

$$I_{CM} = \left(\frac{.05}{8}\right)\left((1.4)(.03) - (.2)(.05)\right)$$

$$= \frac{10^{-4}}{8}\left((1.4)(15) - (.2)\,25 \right)$$

$$= 10^{-4}\,\frac{5}{8}\left((1.4)\,3 - (.2)\,5 \right) = 10^{-4}\,\frac{5}{8}(3.2)$$

$$I_{CM} = 2 \times 10^{-4}\ kg\,m^2$$

Rolling: cont.

$$I_{contact} = I_{cm} + MR^2$$

$$= 2 \times 10^{-4} + (.2)\, 25 \times 10^{-4}$$

$$I_{cont.} = 7 \times 10^{-4} \text{ kg m}^2$$