1) A moving electron of mass m_e and initial speed v_1 collides head on with an initially stationary Hydrogen atom of mass m_p. The collision is elastic and remains 1-dimensional. If $m_p / m_e = 1840$:

(a) What is the center of mass velocity?

(b) What does this collision look like from the center of mass reference frame? Draw two pictures (before and after).

(c) what fraction of the electron's initial kinetic energy is transferred to the Hydrogen atom ?

2) A certain spring is found not to conform to Hooke's law. The return force it exerts on being extended a distance x is found to be $F = -(k_1x + k_2x^3)$.

a) Compute the potential energy function for this force.

b) If a mass m is attached to the spring and stretched out to a distance x_o, what speed will it have on release when it is at the position $x = x_o/2$?

3) A stone of weight w is thrown vertically upward into the air from ground level with initial speed v_0. If air drag exerts a force of constant magnitude f on the stone throughout its flight:

a) what is the maximum height reached by the stone ?

b) what is the return speed just before impact on the way down ?

4) A 1400 kg cannon fires a 70 kg shell with a muzzle speed of 556m/s.If the cannon is mounted on frictionless rails and set at an elevation of 39 degrees above the horizontal:

a) What is the recoil speed of the cannon?

b) At what angle with respect to the ground is the shell projected?

5) Newton's second law is best stated in terms of momentum because in that form variable mass problems can be simply treated.

a) State the law in that form.

b) Now consider the following: a limp flexible chain of total mass M and length L is held at rest by one end so that it is dangling over a smooth floor. The lower end of the rope just grazes the floor. We now release our grasp and allow the chain to fall straight down. As more and more chain piles up on the floor an ever greater normal force must be applied by the floor. Find the required normal force at the moment when half the chain is left in the air and half lies on the floor. (Hint: in time dt how much mass enters the pile on the floor and how fast is it going?)

I. KINEMATICS / BALLISTICS / DIMENSIONS

1) A child in danger of drowning in a river is being carried downstream by a current that flows uniformly with a speed of 2.5 km/h. The child is .6 km from shore and .8 km upstream of a boat landing when a rescue boat leaves the landing to rescue the child. The boat is able to proceed at a maximum speed of 20 km/h with respect to the water.

(a) At what angle with respect to the shore should the axis of the boat be set ?

(b) At what angle with respect to the shore will the resultant *velocity* of the boat be ?

(c) How long will it take to reach the child ?

2) Suppose we fire a sphere of mass M horizontally off a table top so that it lands on the floor some distance away. If the sphere is launched from a vertical height H above the floor and achieves a horizontal range X. Use these two measured numbers and kinematics to:

(a) Derive from fundamental relations what its time of flight must have been.
(b) Derive from fundamental relations what its initial velocity must have been.
(c) With what velocity does it strike the floor and with what speed is it traveling?

3) A wheel 0.75 m in radius rotates freely on its axle at a constant rate of 300 revolutions per minute
(a) Find the speed and acceleration of a stone lodged in the tread of the wheel.
(b) Draw a careful diagram of the situation and draw in the position, velocity and acceleration vectors for the stone at any one given moment.
(c) Now suppose the wheel is placed on the road and allowed to roll forward while revolving at this rate. Find the velocity and acceleration of the tire's center and of the stone at the two moments it is either on the top or bottom of its ride.

4 & 5) Express the *dimension* of each of the following in terms of the **three basic S.I. ones *L, T, M***: {displacement, area, volume, Velocity, Acceleration, Momentum, Force, Torque, Energy, Power, speed, Tension, Angular Momentum, Impulse, Work, Mass, angle, angular velocity, angular acceleration }

II. BASIC DYNAMICS

6) A rope of length **R** has one end tied to a fixed point and the other end tied to a pail full of water. The pail of water of mass **M** is brought into vertical circular motion. The rope can sustain a tension of $\mathbf{T_0}$ and no more before breaking.
a) What is the minimum speed of the pail at the top of the circle if no water is to spill out? What is the tension in the rope at that point?
b) If the pail is in uniform circular motion, what is the minimum speed at which the rope will break and at what point in the circle will it break?

7) A bullet of mass 4 g and a speed of 500 m/s strikes a tree and penetrates to a depth of 6cm .
a) Use the work-energy theorem to find the average frictional force that stops the bullet..
b) Assuming the frictional force is constant, determine how much time elapses while the bullet is decelerating in the tree.

III. ENERGY

8) How do you know if a given force is conservative? _Carefully_ explain how you would go about defining a potential function for a given conservative force (_be sure_ to explain briefly the significance of the choice of reference point).

9) Imagine that a *variable* force is applied to a mass m which is moving on a given *curved* path.

a) Write down an Integral expression for the total work done by $\vec{\mathbf{F}}$ on m.

b) The usefulness of the concept *kinetic energy* rests on the Work-Energy theorem. State the theorem carefully for the general 3-D case.

10) A 5kg block is moving along a frictionless horizontal surface toward a spring with force constant k = 500N/m attached to a wall as shown. Its initial velocity is 6 m/s .

a) Find the maximum compression of the spring when the block hits it.

b) If we wished the maximum compression to be just half as much, what initial velocity should we use?

11) The gravitational potential energy of a mass m at a distance r from the earth's center is given by the function -mMG / r .

a) Use this (as was done in class) to derive the escape velocity for a projectile fired from the earth's surface.

b) EXTRA CREDIT: If the moon's mass is 1/81 of the earth's mass and its radius is 3/11 of the earth's radius, what is the escape velocity from the moon's surface *compared* to that from the earth ?

IV. MOMENTUM/CENTER OF MASS

12) **a)** Give the precise formal definition of the center of mass position (in symbols).

b) Three point masses of mass values 2kg, 2kg, and 4kg are placed on the vertices of a massless equilateral triangle of side length 2m. Find the center of mass position. (as was done in class).

13) A 5 gram bullet is fired horizontally into a 1.2 kg wooden block resting on a rough horizontal surface. The coefficient of friction between the block and the surface is .2 . The bullet remains embedded in the block which is observed to slide .23 meter before coming to rest. What was the initial speed of the bullet?

14) Block A has mass 1kg and block B has mass 3kg. They stand at rest side by side on a frictionless horizontal surface with an ideal massless spring compressed between them. The spring is now released and it pushes the blocks apart. Block B acquires a speed of 1.2 m/s .

a) What is the speed acquired by block A?

b) How much energy was stored in the compressed spring?

c) What is the final speed of the center of mass of the system?

15) A moving electron of mass m_e and initial speed v_1 collides head on with an initially stationary Hydrogen atom of mass m_p. If the collision is ***elastic*** and 1-dimensional and if $m_p / m_e = 1840$:

(a) What is the center of mass velocity?

(b) What does this collision look like from the center of mass reference frame? Draw *two* ***CAREFUL*** pictures (*before* and *after*) to depict the collision from this frame.

(c) When viewed _entirely_ from the _center of mass_ frame, what fraction of the electron's initial kinetic energy is transferred to the Hydrogen atom ?

(d) When viewed from the original LAB frame, what final velocity does the Hydrogen atom recoil with? (**don't** use "canned formulas" here ... you won't need them!)

V. ROTATION / EQUILIBRIUM/ OSCILLATION

16) Suppose that we hang from its rim a hoop of radius 30cm and mass .4kg and set it swinging. (an example would be a spare bicycle wheel hung on the wall by its rim over a nail in the garage)
a) Define the moment of inertia formally and find it for the given hoop.
b) State the fundamental dynamical equation of rotational motion.
c) Now apply the equation to this problem carefully inserting all the specifics as far as you know them.
d) In the limit of *small displacements* solve the equation of motion and identify the period of the motion.

18) Conditions of Static Equilibrium: A ladder of mass **M** and length **L** leans up against a building. A man of mass **m** stands at the midpoint of the ladder. The building has a smooth wall and supplies no frictional force to the ladder so that only the frictional force of the ground on the ladder keeps its foot from slipping out. If the ladder makes an angle θ with the building, what will be the minimum necessary coefficient of static friction such that the ladder will stand without slipping ?

VII. A PROBLEM

20) A child of mass **M** runs at speed **v** up to a flat circular merry-go-round of equal mass **M** and radius **R** and jumps aboard. If the child wishes to bring about a maximum resultant angular velocity, where and in what manner should she jump aboard? What will be the resultant angular velocity? {***Careful!*** She has to balance the added inertia she adds (which slows the resulting angular velocity) against the added lever-arm (which enhances the angular velocity)!}

*A hoop of radius R and mass M rolls without slipping down an inclined plane. Assume the plane is inclined at an angle α and the hoop starts from rest. We analyze this system in problems **1) and 2)** .*

1)

(a) Apply Newton's second law for rotation to find the resulting rotational acceleration.
(b) From your result above find the linear acceleration of the center of mass .
(c) What is its center of mass velocity after rolling a distance d ?

2)

(a) Examine the center of mass motion to find the frictional force on the hoop.
(b) What is the minimum coefficient of static friction which will produce this force?

*A simple uniform rod of mass **M** and length **L** hangs freely at rest from a frictionless pivot at the upper end (i.e. vertically). We wish to use it as a **Ballistic Pendulum**. A blob of putty of mass **m** will be shot horizontally at a spe*
v *so that it collides with and sticks to the lower hanging end of the rod at its lowest point. (this is quite like what w actually performed in lab !) We analyze this system in problems **3) and 4)***

3) Our first task is to find the ***Moment of Inertia*** of the rod-putty system described above by treating it as a ***Physical Pendulum***. Then we will find the ***Center of Mass***. Please complete the following steps:

(a) What is the ***Moment of Inertia*** of the hanging rod alone (in terms of M and L) ?
(b) What ***Moment of Inertia*** is added to the rod when the putty sticks to the lower hanging end of the rod at its lowest point (in terms of M, m and L)?
(c) Now where is the ***Center of Mass*** of the rod-putty system?

(d) ***Carefully*** state Newton's second law for angular motion *as it applies* to this case.
(c) For small angular motion, show that the system will exhibit ***Simple Harmonic Motion***.
and find the predicted ***Frequency*** of the resulting motion in terms of (m, M, L, and g)

4) Now we shoot the putty horizontally and at speed **v** at the rod so that it sticks to the lower hanging end! Analyz the collision and subsequent motion of the pendulum using *whatever physical principles hold true* (as we did in lab
(a) What is the ***angular velocity*** of the hanging rod–putty system just after the collision of the putty with the rod ?
(b) Now find the maximum angle reached by the pendulum in terms of (I, R_{cm}, M_{tot}, m, **v**, L)

5) STATICS: A ladder of mass M and length L leans up against a building. The building has a smooth wall and supplies no frictional force to the ladder so that only the frictional force of the ground on the ladder keeps its foot from slipping out. If the ladder makes an angle **θ** with the building:

(a) What are the general (abstract) conditions on a rigid object that it be in static equilibrium?
(b) Apply the general conditions of equilibrium to the specifics of this case.
(c) Now find the **minimum necessary coefficient of static friction** such that the ladder will stand without slipping

1) A uniform solid disk of mass M and radius R rolls without slipping down an inclined plane having started from rest. The plane's angle of inclination is specified as θ. Working from fundamental relations only, please find:
(a) the acceleration of the disk's center of mass.
(b) the direction and magnitude of the frictional force applied by the plane on the disk.

2) A uniform rod of length L and mass M is free to rotate on a frictionless pivot through one end. It is subject to the force of gravity and the forces from the pivot.
(a) Define in symbols (an integral) what we *mean* by moment of inertia and **derive** from the definition the moment of inertia of the rod about its pivot.
(b) State Newton's second law for angular motion as it applies to this case (you will need to introduce a descriptive angle).
(c) State the parallel axis theorem (in symbols) and use it to relate the moment of inertia of the rod about one end to that it would have if pivoted about its center.

3) Reconsider the pivoted rod situation in problem **2)** . If it starts to fall sideways from an upright vertical position (having been initially at rest):
(a) What is its angular acceleration and angular velocity at the moment it is horizontal ?
(b) What are the components of the reaction force of the pivot on the rod at the moment it is horizontal ?
(c) Suppose now that we were to remove the pivot and, once again starting from a vertical position, allow the lower end to rest on a frictionless smooth surface. How will the descent qualitatively differ from that of part a) and:
EXTRA CREDIT, what is now its angular acceleration and angular velocity at the moment it is horizontal ?

4) A girl of mass M stands on the rim of a frictionless merry-go-round of radius R and rotational inertia I. They are both at rest. She now throws a rock of mass m horizontally off of the merry-go-round and at speed v_o.

(a) How might she throw the rock so as to minimize the rotation induced in the merry-go round (draw a picture) ?
(b) How might she throw the rock so as to maximize the rotation induced in the merry-go round (draw a picture) ?
(c) In this second case what will be the resulting rotational speed of the merry-go round?

5) Ballistic Pendulum. Our Ballistic Pendulum experiment in lab was treated as a conservation of linear momentum experiment. In hindsight we can see that with the swinging arm equipment we used this isn't rigorously correct.

(a) Argue instead that linear momentum couldn't possibly have been conserved but that instead it is *angular* momentum which, in fact, is conserved.

(b) If, then, a projectile of mass m and initial speed v_o is fired horizontally at the lower end of our freely hanging Ballistic Pendulum and sticks there, derive what the linear speed of the ball will truly be immediately after the collision. Assume that the Ballistic Pendulum has length L and rotational inertia I.

1) A moving particle of mass m_1 and initial speed v_1 collides head on with an initially stationary particle of mass m_2. A massless spring of spring-constant **k** has been attached to the near side of mass m_2 and so the collision is elastic (m_1 hits the spring compressing it) and the motion throughout remains 1-dimensional.

(a) Define (in symbols) what we mean by the center of mass of this system and state what the motion of the center of mass of this system is.

(b) What does this collision look like from the *center of mass* reference frame?

(c) Work from *basic conservation laws* to derive the resulting final velocity of each mass *as it appears in the center of mass frame* .

(d) What is the maximum compression of the spring?

2) A hunting rifle fires a bullet of mass 0.012kg with a muzzle velocity of 600 m/s. The rifle has a mass of 4kg.

(a) What is the recoil velocity of the rifle as the bullet leaves the barrel ?

(b) If the rifle is stopped by the hunter's shoulder in a distance of 2.5 cm, what is the average force on the hunter's shoulder ?

(c) In this case, then, how much time is required to stop the rifle?

(d) If the hunter were to brace his shoulder so the recoil distance were shorter, how would your answers to the above change ?

3) A block of mass m starts from rest and slides down a rough plane inclined at angle α . If the coefficient of kinetic friction is μ, what will be the speed of the mass after having traveled a distance **s** along the incline ?

4) Imagine that a variable force $\vec{\mathbf{F}}$ is applied to a mass m which is moving on a **given** *curved* path.

a) Write down an expression for the total work done by on m between any two points of the path.

b) The usefulness of the concept *kinetic energy* rests on the Work-Energy theorem. State the theorem carefully for the general **3-D** case.

c) How do you know if a given force is conservative (what explicit experiments would you have to conduct)?

d) Carefully explain how you would go about defining a potential function for a given conservative force (be sure to explain briefly the significance of the choice of reference point).

5) Newton's second law is best stated in terms of momentum because in that form *variable mass* problems can be simply treated.

a) State the law in that form.

b) Now consider the following: a limp flexible chain of total mass M and length L lies coiled on a smooth floor. We now grasp one end and apply sufficient force to lift the end at constant speed $\mathbf{v_o}$ straight upward. As more and more chain unwinds and is lifted we must apply ever more force. Find the required force when we have lifted the end to any given height *h*.

4) A stone of weight w is thrown vertically upward into the air from ground level with initial speed v_0. If air drag exerts a force opposed to the motion of *constant magnitude* **f** on the stone throughout its flight:

a) what is the maximum height reached by the stone ?

b) what is the return speed just before impact on the way down ?

4) A 1400 kg cannon fires a 70 kg shell with a muzzle speed of 556m/s.If the cannon is mounted on frictionless rails and fired horizontally:

a) What is the recoil speed of the cannon?

b) What is the ratio of the cannon's kinetic energy to the shell's?

c) If the cannon is brought to rest in 2m by a constant resisting force F_r, what value must that force have?

d) How much time, then, does it take to stop the cannon?

In the diagram (2) below we observe a 3 kg block resting on a rough surface with μ_k =.15 . It is connected to a hanging 2 kg block by a frictionless pulley. Use ENERGY CONCEPTS to find the speed of the hanging block just before hitting the floor 1.5m below.

4) A 5 gram bullet is fired horizontally into a 1.2 kg wooden block resting on a rough level surface. The coefficient of friction between the block and the surface is .2 . The bullet remains in the block which then slides .23 meter before coming to rest.

a) What was the initial speed of the bullet?

b) What net ***work*** is done by the surface's frictional force ?

c) What ***impulse*** is delivered by the surface's frictional force?

d) What ***power*** is delivered by the surface's frictional force?

5) **a)** The usefulness of the concept *kinetic energy* rests on the Work-Energy theorem. ***Derive*** the theorem carefully for the general 3-D case.

b) **Show explicitly** from ***Newton's laws*** that when two particles collide, that we may conclude momentum is conserved *moment by moment* throughout the collision process and from this, prove that the **center of mass** has constant velocity.

5) **a)** The usefulness of the concept *kinetic energy* rests on the Work-Energy theorem. ***Derive*** the theorem carefully for the general 3-D case.

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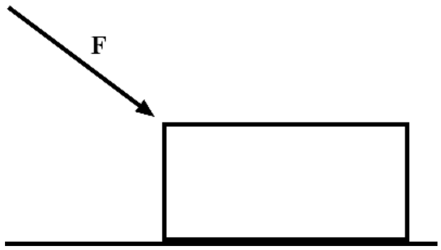
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