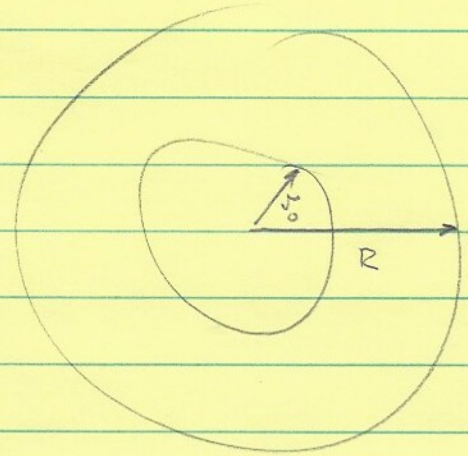
Set 13   Chap. 12 Knight   1/

1.) #55 pg 332

a)



$$I = \int dm\, r^2 \qquad \frac{dm}{M} = \frac{dA}{A} = \frac{2\pi r\, dr}{\pi (R^2 - r_0^2)} = \frac{dr^2}{R^2 - r_0^2}$$

$$I = \int_{r_0}^{R} M \frac{dr^2}{R^2 - r_0^2} r^2 = \frac{M}{R^2 - r_0^2} \int_{r_0^2}^{R^2} r^2\, dr^2 = \frac{M}{2} \frac{R^4 - r_0^4}{R^2 - r_0^2}$$

$$I_{cm} = \frac{M}{2}\left(R^2 + r_0^2\right)$$

b) If $r_0 \to 0$ we recover $\frac{1}{2} M R^2$ ✓ solid disk
If $r_0 \to R$ we recover $M R^2$ ✓ hoop.

c) $I_{tot}\alpha = \tau = MgR\sin\theta \rightarrow I_{tot}\,\alpha R = Mg R^2 \sin\theta$

so $a_{cm} = \alpha R = \frac{MR^2 \sin\theta}{I_{tot}} g = \frac{MR^2}{M\frac{(R^2+r_0^2)}{2} + MR^2} g \sin\theta$

$$a_{cm} = \frac{g\sin\theta}{\frac{3}{2} + \frac{1}{2}\frac{r_0^2}{R^2}} = \frac{2g\sin\theta}{3 + \left(\frac{1.5}{2}\right)^2} = .561\, g\sin\theta$$

$v_f^2 - \cancel{v_i^2}^{\,0} = 2a_{cm}(.5\,m)$ so $v_f^2 = 2a_{cm}(.5)$

cont. 1.) for frictionless slide

$$v_{f_0}^2 = 2\, g \sin\theta\, (.5)$$ so ...

rolling $\quad v_{f_r}^2 = 2(.561\, g \sin(20^\circ))(.5\,m) \Rightarrow v_f = 1.37\,\frac{m}{s}$

sliding $\quad v_{f_0}^2 = 2(g \sin(20^\circ))(.5\,m)$

$$\frac{v_{f_r}}{v_{f_0}} = \sqrt{.561} = .749$$

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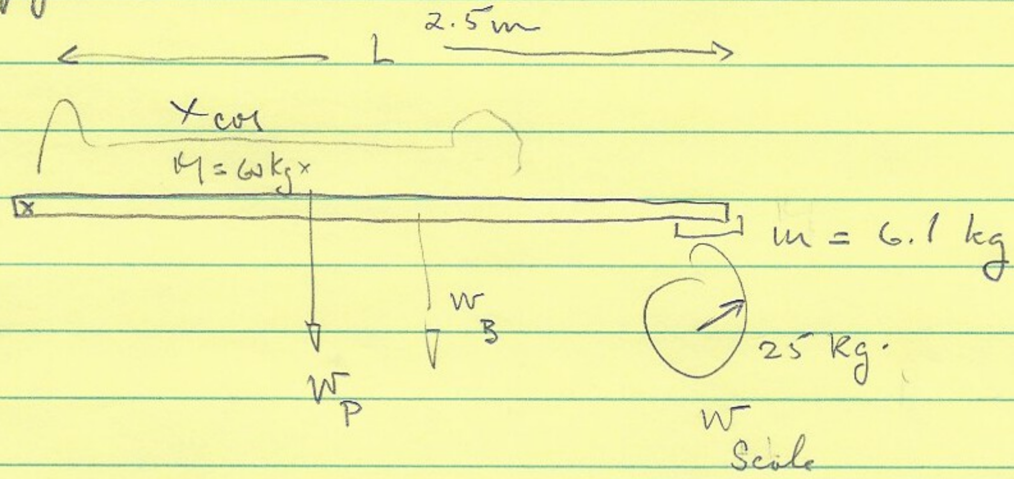
2.) # 57 pg 333



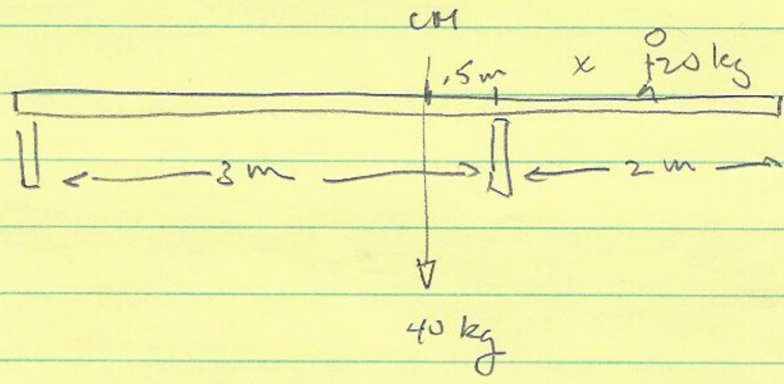
$$W_P X_{CM} + W_B \frac{L}{2} = W_{Scale} L$$

$$\Rightarrow \quad \frac{X_{CM}}{L} = \frac{-\frac{1}{2} W_B + W_{Scale}}{W_P} = \frac{-\frac{1}{2}(6.1) + 25 \text{ kg}}{60}$$

$$\frac{X_{CM}}{2.5} = .3658 \quad \text{so} \quad X_{CM} = .9146 \text{m}$$

Set 12 Knight Chap 12

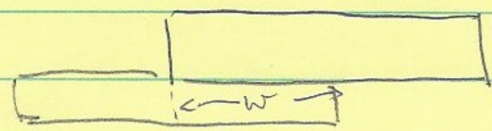
3.) #60

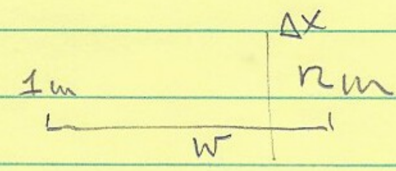


$$(40\ kg)(.5) = (20\ kg)\ x_c$$

$$1\ m = x_c$$

4) #61





cm $\Delta x = \frac{1}{n+1} w$

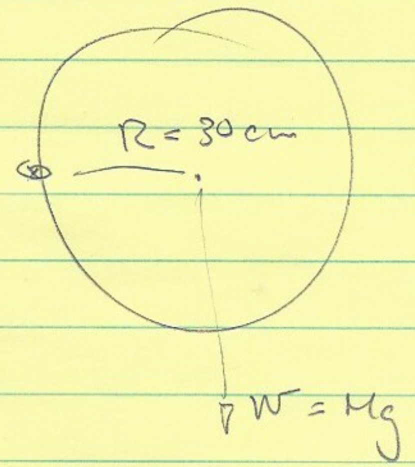
for us, $w = \frac{L}{2}$ use four bricks

total distance $= \frac{L}{2}\left\{ 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right\}$

$= L\,\frac{25}{24} > L$!

$\sum_{n=1}^{\infty} \frac{1}{n} = \infty$! No limit !

5.) # 72 pp 334



$\tau = I_{tot}\alpha$

$MgR = (\frac{1}{2}MR^2 + MR^2)\,\alpha$

$\alpha_{init} = \frac{2}{3}\frac{g}{R}$

U lost $= MgR$ $\Delta U = \Delta KE$

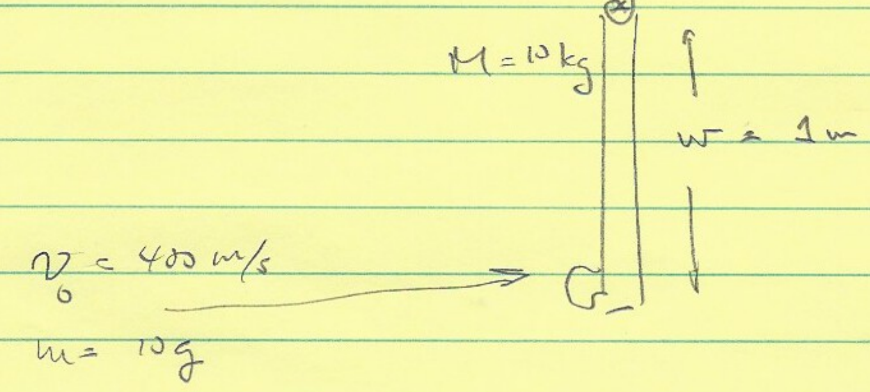
KE gained $= \frac{1}{2} I \omega_f^2 = \frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega^2$

So $\frac{3}{4} R^2 \omega^2 = gR$ $\omega_f^2 = \frac{4}{3}\frac{g}{R}$

$\omega_f = \sqrt{\frac{4}{3}\frac{9.8}{.3}} = 6.6$ rad/sec

6.) # 78 pg 334

top view



$$L_{in} = m v_0 w$$

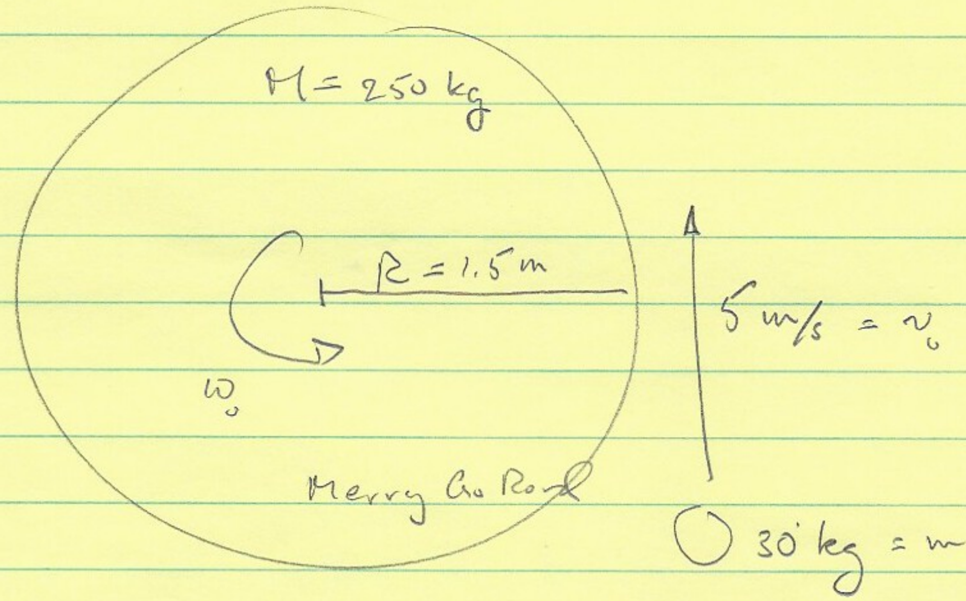
$$L_{out} = I_{tot}\,\omega = \left(\tfrac{1}{3} M w^2 + m w^2\right)\omega$$

So! $m v_0 w = \left(\tfrac{1}{3} M + m\right) w^2 \omega$

$$\Rightarrow \frac{m}{\left(\frac{1}{3} M + m\right)} \frac{v_0}{w} = \omega_{init}$$

$$\omega_{init} = \frac{3(.01)}{10.03} \frac{4 \times 10^2}{1\,m} = 1.196 \text{ rad/sec.}$$

7.) #81 pg 334



$$\omega = 2\pi \frac{(20)}{60\,sec} \text{ rad} = \frac{2\pi}{3} \text{ rad/sec}$$

Cons. of angular momentum!

$$L_f = L_{MGR} + L_{John}$$

$$I_{tot}\,\omega_f = I_{MGR}\,\omega_0 + m v_0 R$$

$$\omega_f = \frac{I_{MGR}\,\omega_0 + m v_0 R}{I_{MGR} + m R^2}$$

$$I_{MGR} = \tfrac{1}{2} M R^2$$

7.) cont. So! $\omega_f = \dfrac{\omega_0 + \dfrac{m v_0 R}{\frac{1}{2} M R^2}}{1 + \dfrac{m R^2}{\frac{1}{2} M R^2}}$

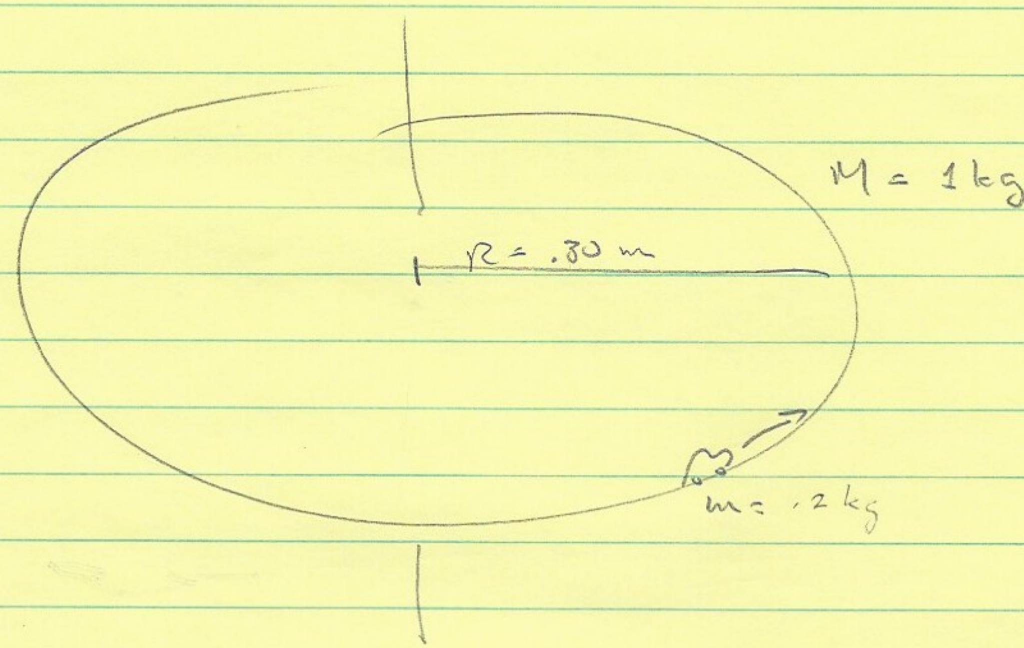
$$= \frac{\omega_0 + \frac{2m}{M}\frac{v_0}{R}}{1 + \frac{2m}{M}}$$

$$= \frac{\frac{2\pi}{3} + \frac{60}{250}\frac{5}{1.5}}{1 + \frac{60}{250}} = \frac{\frac{2\pi}{3} + \frac{4}{5}}{1 + \frac{6}{25}} = 2.334 \frac{\text{rad}}{\text{sec}}$$

$$\frac{\text{rad}}{\text{sec}} = \frac{1}{2\pi}\frac{\text{rev}}{\frac{1}{60}\text{min}} = \frac{60}{2\pi}\frac{\text{rev}}{\text{min}} = \frac{30}{\pi}\frac{\text{rev}}{\text{min}}$$

Ans. 22.29 rev/min

8.) #88 pg 335



No Angular momentum! $L_{car} + L_{track} = 0$

$$\left.\begin{aligned} L_{car} &= m v_c R \\ L_{track} &= I\,\omega_t \end{aligned}\right\} \text{ but } \begin{aligned} I\omega_t &= -m v_c R \\ I &= MR^2 \end{aligned}$$

We are told $v_c - \omega_t R = .75$ m/s

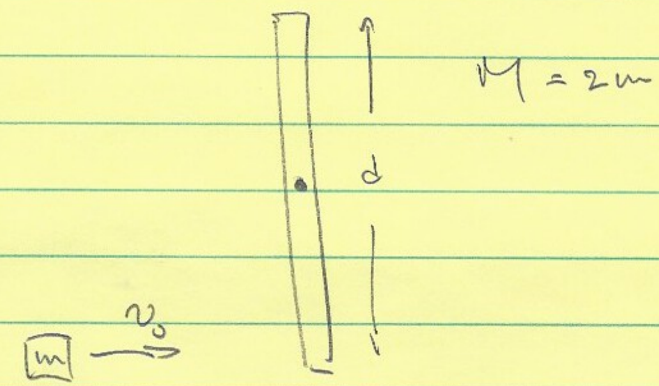
$$\Rightarrow v_c = -\frac{I\omega_t}{mR} \Rightarrow -\frac{I\omega_t}{mR} - \omega_t R = .75 \text{ m/s}$$

$$\omega_t\left(\frac{I}{mR} + R\right) = -.75 \text{ m/s}$$

$$\omega_t = \frac{(-.75 \text{ m/s})/R}{\frac{M}{m} + 1} = -.41\bar{6} \text{ rad/sec}$$

$$= -3.98 \text{ rev/min}$$

9.) # 89 pp 335



✓ Angular momentum is conserved
✓ KE. is preserved.

$$L_{m_i} = L_{m_f} + L_{R_f}$$

$$\frac{L^2_{m_i}}{2I_m} = \frac{L^2_{m_f}}{2I_m} + \frac{L^2_{R_f}}{2I_R}$$

So! $\frac{1}{2I_m}\left(L^2_{m_i} - L^2_{m_f}\right) = \frac{1}{2I_R} L^2_{R_f}$ divide!

$$\left(L_{m_i} - L_{m_f}\right) = L_{R_f}$$

$$\frac{1}{2I_m}\left(L_{m_i} + L_{m_f}\right) = \frac{1}{2I_R} L_{R_f}$$

$$\begin{cases} L_{m_i} + L_{m_f} = \frac{I_m}{I_R} L_{R_f} \\ L_{m_i} - L_{m_f} = L_{R_f} \end{cases}$$

$$2 L_{m_i} = \left(1 + \frac{I_m}{I_R}\right) L_{R_f}$$

or $$\frac{2}{1 + \frac{I_m}{I_R}} L_{m_i} = L_{R_f}$$

and $$2 L_{m_f} = \left(\frac{I_m}{I_R} - 1\right) L_{R_f}$$

$$\cancel{2} L_{m_f} = \cancel{2} \frac{\left(\frac{I_m}{I_R} - 1\right)}{\left(\frac{I_m}{I_R} + 1\right)} L_{m_i}$$

So! $$L_{R_f} = \frac{2}{1 + \frac{I_m}{I_R}} L_{m_i}$$

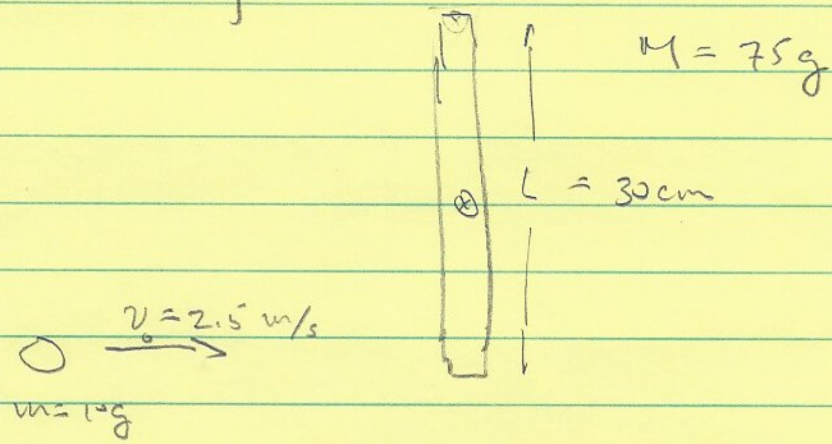
$$L_{m_f} = \frac{\left(\frac{I_m}{I_R} - 1\right)}{\left(\frac{I_m}{I_R} + 1\right)} L_{m_i}$$

$$\frac{I_m}{I_R} = \frac{m\left(\frac{d}{2}\right)^2}{\frac{1}{12} 2m\, d^2} = \frac{12}{8} = \frac{3}{2}$$

$$v_f = \frac{\frac{3}{2} - 1}{\frac{3}{2} + 1} v_i = \frac{\frac{1}{2}}{\frac{5}{2}} = \frac{1}{5} v_i$$

Set 12 Knight Chapter 12

10.) #90 pg 335 Before



Physics?

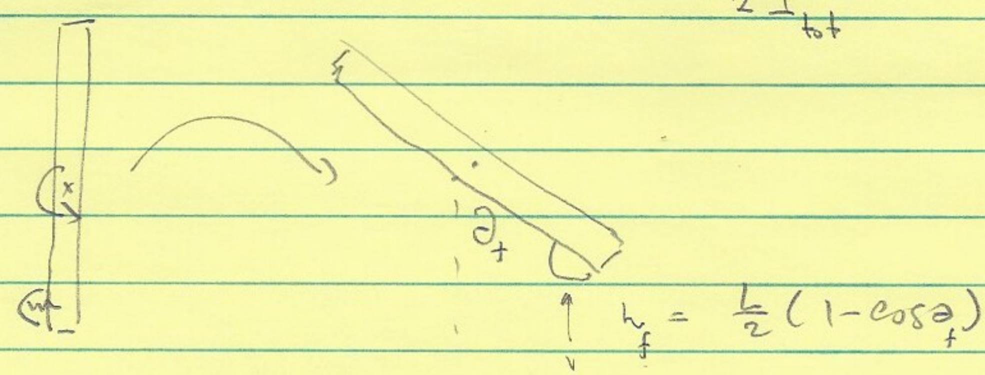
$$L_{in} = L_{out} \rightsquigarrow m v_0 \frac{L}{2} = I_{tot}\,\omega_0$$

$$I_{tot} = I_{rod} + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}mL^2$$

just following the collision...

$$\text{total kinetic energy out} = \frac{L_{out}^2}{2I_{tot}} = \frac{L_{in}^2}{2I_{tot}}$$

After



$$(KE)_i \rightsquigarrow (PE)_f \quad \text{so} \quad \frac{L_{out}^2}{2I_{tot}} = mg\frac{L}{2}(1-\cos\theta_f)$$

so!

$$\frac{\left(m v_0 \frac{L}{2}\right)^2}{2\left(\frac{1}{12}ML^2 + \frac{1}{4}mL^2\right)} = mg\frac{L}{2}(1-\cos\theta_f)$$

Set 12 Knight

cont. 10.)

$$\frac{\frac{v_0^2}{gL}}{4\left(\frac{1}{12}\frac{M}{m}+\frac{1}{4}\right)} = 1-\cos\theta_f$$

$$\cos\theta_f = 1 - \frac{v_0^2/gL}{\frac{1}{3}\frac{M}{m}+1}$$

$$= 1 - \frac{(2.5)^2/(9.8)(.3)}{\frac{1}{3}\left(\frac{75}{10}\right)+1}$$

$$= 1 - .607 = .3926 \Longrightarrow \theta_f = 66.88^\circ$$