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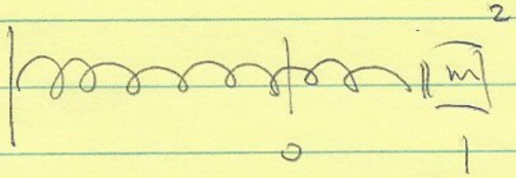
1.) #16 pg 415



$f = 2\ Hz$

$\omega = 4\pi$ rad/sec.

$x = 5\ cm$

$v_x = -30\ cm/s$

We know ✓ $a = -\omega^2 x$

✓ $v^2 + \omega^2 x^2 = v_{max}^2 = \omega^2 x_{max}^2$

a) $f = 2$ cycles/sec $= \frac{1}{T}$ so $T = \frac{1}{2}$ sec/cycle

b) $\omega = 2\pi f = 4\pi$ rad/sec.

c) $v^2 + \omega^2 x^2 = \omega^2 x_{max}^2$ so $x_{max}^2 = x^2 + \frac{v^2}{\omega^2}$

$x_{max}^2 = (5cm)^2 + \left(\frac{30\ cm/sec}{4\pi\ rad/sec}\right)^2$ so $x_{max} = 5.54\ cm$

d) Use $x_0 = A\cos\varphi_0$ & $v_0 = -\omega A\sin\varphi_0$

so $\frac{v_0}{x_0} = -\omega\tan\varphi_0$ or $\tan\varphi_0 = -\frac{v_0}{\omega x_0}$

$\tan\varphi_0 = \frac{-30\ cm/s}{4\pi \frac{rad}{sec} 5\ cm} = -.477$ $\varphi_0 = -25.5^\circ$

$= -.445$ rad.

e) $v_{max} = \omega x_{max} = 4\pi(5.54) = 69.6$ cm/sec.

f) $a_{max} = -\omega^2 x_{max} = 874.8\ cm/sec^2$

g) $E = \frac{1}{2} m v_{max}^2 = 4.8 \times 10^{-2}\ J$ h) $x = x_{max}\cos(\omega t + \varphi_0) = -.723$ cm

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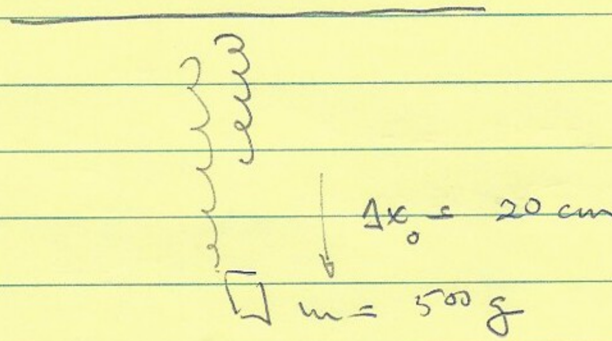
2.) #21 pgs 416



a) since $k \Delta x_0 = mg \Rightarrow k = \frac{mg}{\Delta x_0} = \frac{.5\,(9.8)}{.2}$

$k = 24.5 \ N/m$

b) since $\sqrt{\frac{k}{m}} = \omega = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{.5}{24.5}} = .898 \text{ sec.}$

c.) $v_{max} = \omega x_{max} = \sqrt{\frac{k}{m}}\,(10\text{cm}) = .7 \text{ m/sec}$

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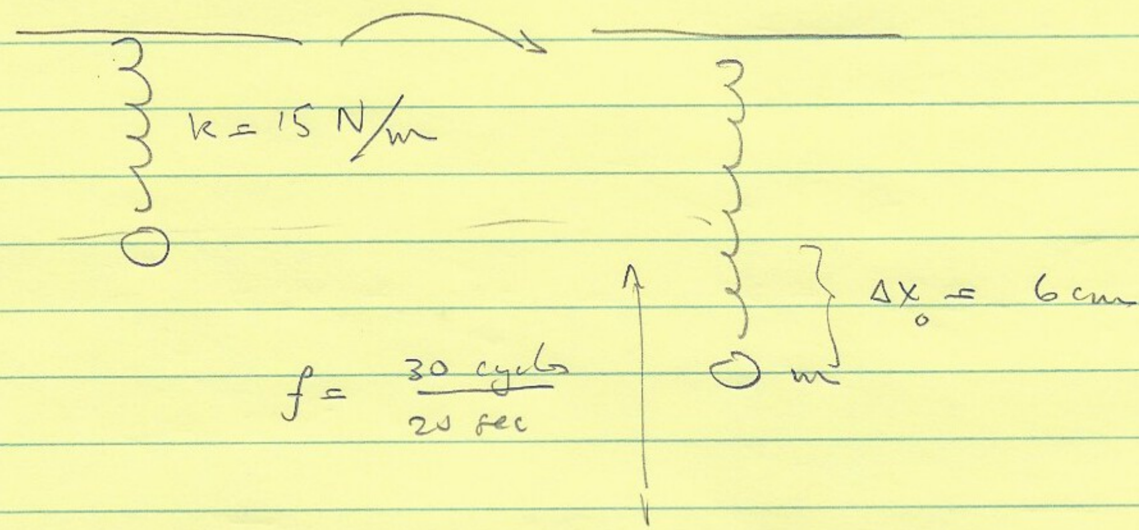
3.) #22 pg 416



a) so $f = 1.5$ cycle/sec. $\Rightarrow \omega = 2\pi\left(\frac{3}{2}\right)$ rad/sec. $= 9.42$ rad/sec

$\omega^2 = \frac{k}{m} \Rightarrow m = \frac{k}{\omega^2} = \frac{15\ N/m}{(9.42)^2} = .169\ kg.$

b) $v_{max} = \omega x_{max} = (9.42\ \text{rad/sec})(6\ cm) = .565\ m/sec.$

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4.) #43 pg 417

Spacing (unit)? , .75 sec. & T = 3 sec., $\omega = \frac{2\pi}{T}$ = 2.

a) if $k = 240$ N/m then $\frac{k}{m} = \left(\frac{2\pi}{T}\right)^2$

$$\Rightarrow k\left(\frac{T}{2\pi}\right)^2 = m$$

$$\left(240\frac{N}{m}\right)\left(\frac{3\,sec}{2\pi}\right)^2 = m = 54.7\text{ kg}$$

b) The zero point is 1 m and x_0 = .4 m

at $x = 1.2$ $x = \frac{x_{max}}{2}$

Since $v^2 + (\omega x)^2 = (\omega x_{max})^2$

$$v^2 = \omega^2 (x_{max}^2 - x^2) =$$

$$v = \omega x_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2} = \omega x_{max}\sqrt{3/4}$$

v = .7255 m/s

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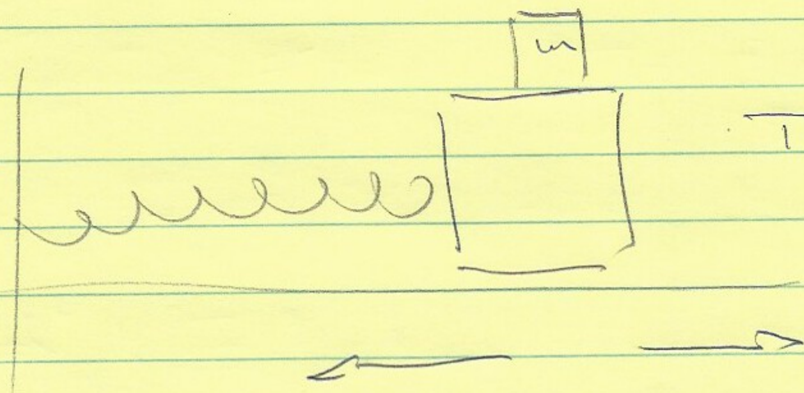
5.) # 52



$T = 1.5$ sec.

slipping starts when $x_0 = .4$ m

$a_{max} = -\omega^2 x_{max}$ but $a = \frac{f_{fric}}{m}$

$a_{max} = \frac{\mu_s mg}{m} = \mu_s g$

$\Longrightarrow \mu_s = \frac{a_{max}}{g} = \frac{\omega^2 x_{max}}{g} = \frac{\left(\frac{2\pi}{3/2}\right)^2 (.4)}{(9.8)}$

$\mu_s = .716$

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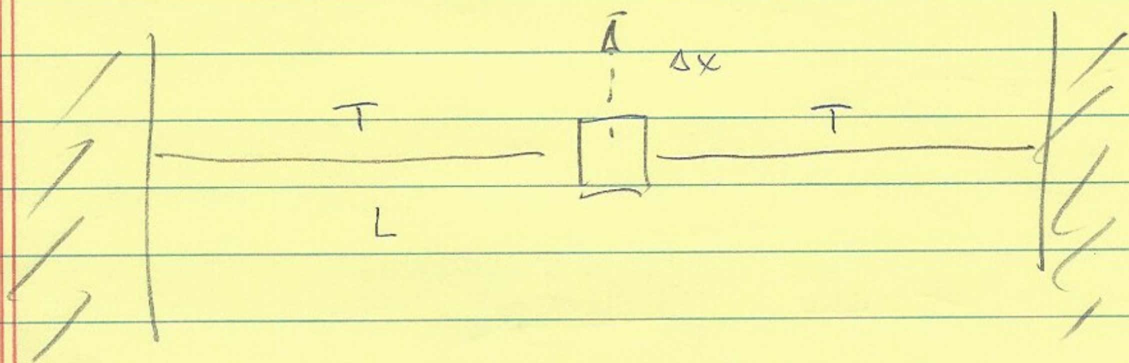
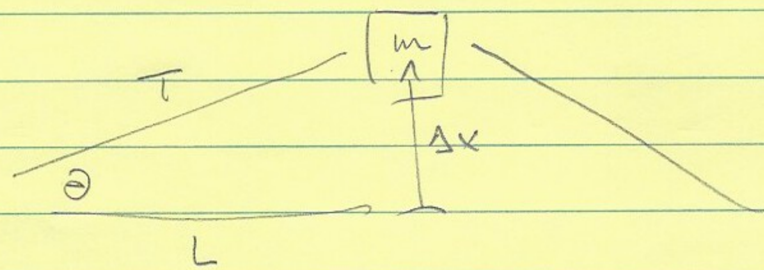
6.) #62 pg 418





Restoring force is $-2T\sin\theta \approx -2T\,\theta = -2T\frac{\Delta x}{L}$

$ma = -2T\frac{\Delta x}{L}$ so! $\omega^2 = \frac{2T}{mL}$

$\Longrightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{2T}{mL}}$

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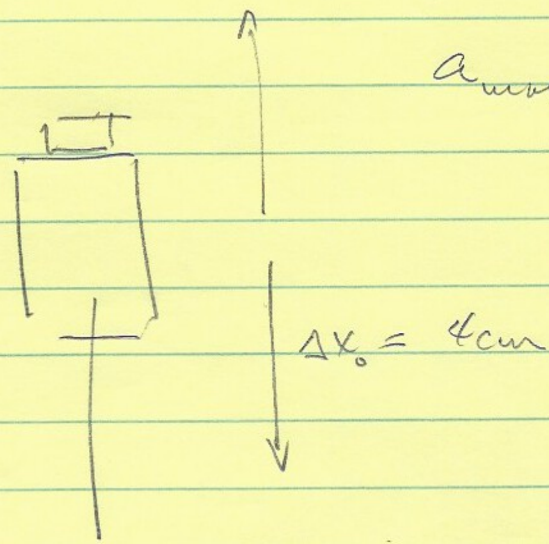
7.) # 64 pg 418

we must have

$a_{max} \leq g$!



a) a_{max} at $x = x_{max} = 4cm$ at the top!

$a_{max} = \omega^2 x_{max} = g$ so! $\omega^2 = \frac{g}{x_{max}}$

$\frac{2\pi}{T} = \sqrt{\frac{g}{x_{max}}} = \sqrt{\frac{9.8}{.04}} = 15.65$ rad/sec

b) $f_{max} = \frac{1}{T} = 2.49$ Hz

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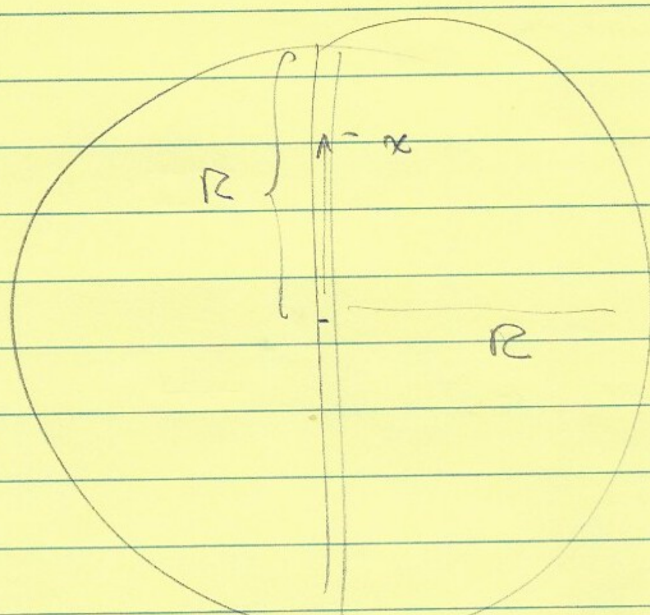
8) # 66 pp 418



The mass out to radius x is: $\frac{M(x)}{M(R)} = \left(\frac{x}{R}\right)^3$

but $g(x) = \frac{M(x)G}{x^2} = \frac{M(R)\left(\frac{x}{R}\right)^3 G}{x^2} = \frac{x}{R}\frac{M(R)G}{R^2}$

$F = mg = m\left(\frac{MG}{R^2}\right)\frac{x}{R}$

$ma = F \Rightarrow a = -\frac{MG}{R^3}x$

$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{MG}{R^3}$

So $\frac{T}{2} = \pi\sqrt{\frac{R^3}{MG}}$

$R = .75 \times 10^5$ m

$M = 3.5 \times 10^{18}$ kg

$G = .667 \times 10^{-10}$

$\frac{T}{2} = \pi\left[\frac{\left(\frac{3}{4}\times 10^5\right)^3}{(3.5\times 10^{18})(.667\times 10^{-10})}\right]^{1/2} = 1.34 \times 10^3$ sec.

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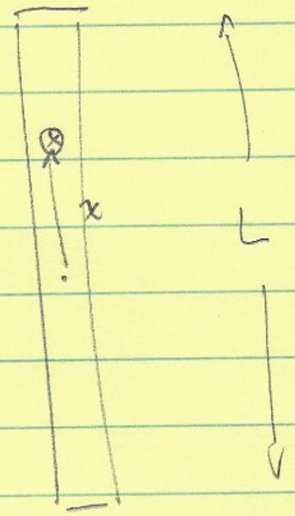
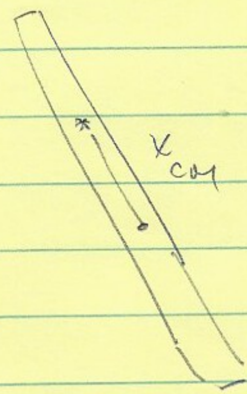
9) #78 pg 419



$$I = I_{cm} + M(x^2)$$

$$= \frac{1}{12} M L^2 + M x^2$$

$$= M L^2 \left(\frac{1}{12} + \left(\frac{x}{L}\right)^2 \right)$$



$$I \alpha = \tau$$

$$I \alpha = -M g x_{cm} \sin\theta$$

small angle $\sin\theta \approx \theta$

so ...

$$\alpha = -\frac{M g x_{cm}}{I} \theta$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{M g x_{cm}}{I} = \frac{M g x_{cm}}{M L^2 \left(\frac{1}{12} + \left(\frac{x}{L}\right)^2\right)}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{L} \frac{(x/L)}{\left[\frac{1}{12} + \left(\frac{x}{L}\right)^2\right]}$$

cont. 9.) # 78 pg 419

$$\frac{T^2}{4\pi^2} = \frac{L}{g}\left[\frac{\frac{1}{12} + \left(\frac{x}{L}\right)^2}{\left(\frac{x}{L}\right)}\right] = \frac{L}{g}\left(\frac{1}{12}\frac{1}{w} + w\right)$$

minimize! $\frac{dT}{dw} \to 0 \Rightarrow \left(\frac{1}{12}\left(-\frac{1}{w^2}\right) + 1\right) = 0$

$w = \frac{x}{L}$ $\qquad \frac{1}{12} = w^2$ or $w = .289$

$$\underline{T_{min}^2} = 4\pi^2 \frac{L}{g}\left(\frac{\sqrt{12}}{12} + \frac{1}{\sqrt{12}}\right)$$

$$= 4\pi^2 \frac{L}{g} \frac{2}{\sqrt{12}} = 4\pi^2 \frac{1}{\sqrt{3}} \frac{L}{g}$$

$$T_{min} = 2\pi \sqrt{\frac{1}{\sqrt{3}} \frac{1m}{9.8\ m/s^2}} = 1.525 \text{ sec.}$$

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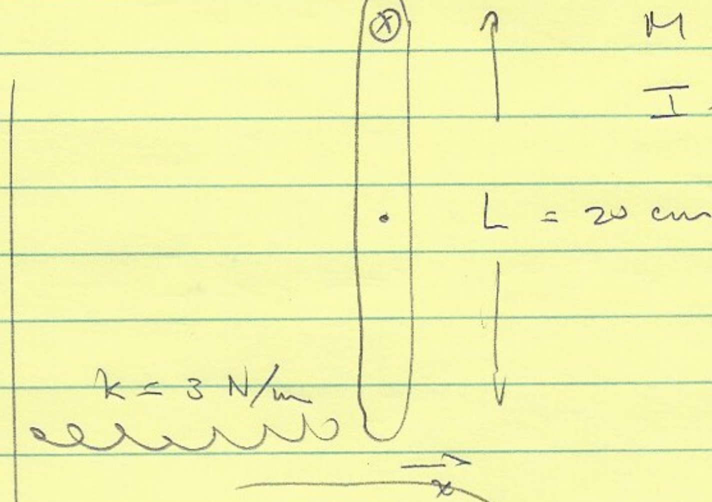
10.) #81 pg 419

$M = 200\,g$

$I = \frac{1}{3} M L^2$



$I \alpha = \tau$

$\tau = ?\ (-kx)L - Mg\frac{L}{2}\sin\theta$

If $\sin\theta \approx \theta$, & $\frac{x}{L} \approx \theta$

$\tau = -\left(kL^2\theta + Mg\frac{L}{2}\theta\right)$

$= -\left(kL^2 + Mg\frac{L}{2}\right)\theta$

So! $\frac{1}{3}ML^2\,\alpha = -\left(kL^2 + Mg\frac{L}{2}\right)\theta$

$\alpha = -3\left(\frac{k}{M} + \frac{g}{L}\frac{1}{2}\right)\theta$

so $\left(\frac{2\pi}{T}\right)^2 = 3\left(\frac{k}{M} + \frac{g}{L}\frac{1}{2}\right) = 3\left(\frac{3}{.2} + \frac{9.8}{.4}\right) = 118.5$

$\frac{2\pi}{T} = 10.88$ $\qquad T = .577$ sec.