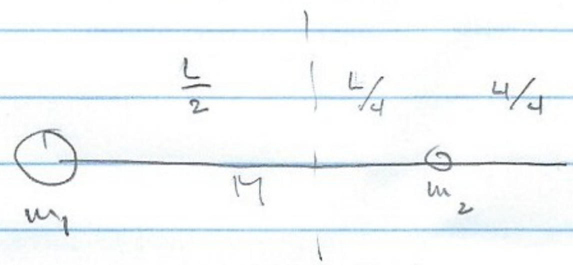
Set 12
Knight
Chap 12

prob 1)

#51 pp 332



Sum the contributing inertias!

rod $\frac{1}{12} M L^2$

m_1 $m_1 \left(\frac{L}{2}\right)^2$

m_2 $m_2 \left(\frac{L}{4}\right)^2$

$$I_{tot} = \left(\frac{M}{12} + \frac{m_1}{4} + \frac{m_2}{16}\right) L^2$$

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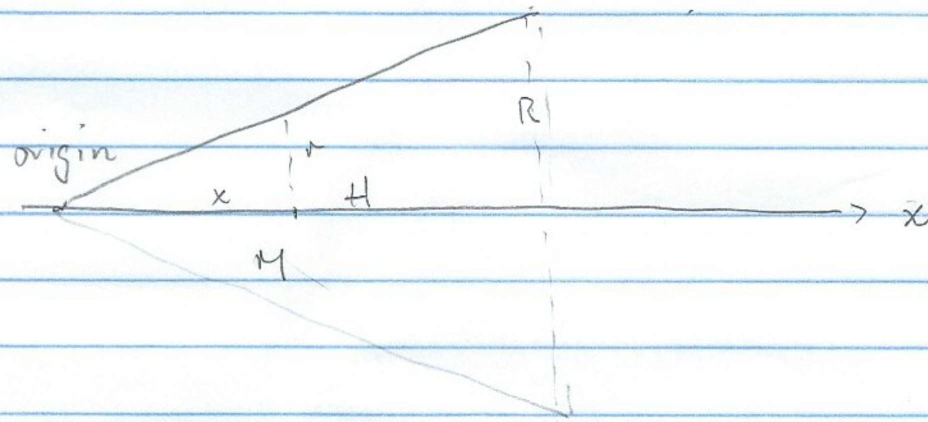
prob 2)

\# 56



volume

$$V_0 = \int dV = \int_0^H \pi r^2 dx \qquad \text{but } \frac{r}{x} = \frac{R}{H} \text{ so } r = x\frac{R}{H}$$

$$V_0 = \int_0^H \pi x^2 dx \left(\frac{R}{H}\right)^2 = \frac{1}{3}\pi H^3 \left(\frac{R}{H}\right)^2 = \frac{1}{3}\pi R^2 H$$

CM.

$$M_0 X_{cm} = \int dM\, x \qquad \text{but } \frac{dM}{M_0} = \frac{dV}{V_0} = \frac{\pi r^2 dx}{\frac{1}{3}\pi R^2 H}$$

$$= 3\left(\frac{r}{R}\right)^2 d\frac{x}{H} \qquad \text{but } \frac{r}{R} = \frac{x}{H}$$

$$\text{so } X_{cm} = H\int 3\left(\frac{x}{H}\right)^2 d\left(\frac{x}{H}\right)\left(\frac{x}{H}\right) = H\frac{3}{4}$$

I.

$$I = \int dI = \int \frac{1}{2} dM\, r^2 = \frac{1}{2} M\, 3\int \left(\frac{r}{R}\right)^2 d\left(\frac{r}{R}\right) r^2$$

$$= \frac{3}{2} MR^2 \int_0^1 \left(\frac{r}{R}\right)^4 d\left(\frac{r}{R}\right) = \frac{3}{10} MR^2$$

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prob. 3)

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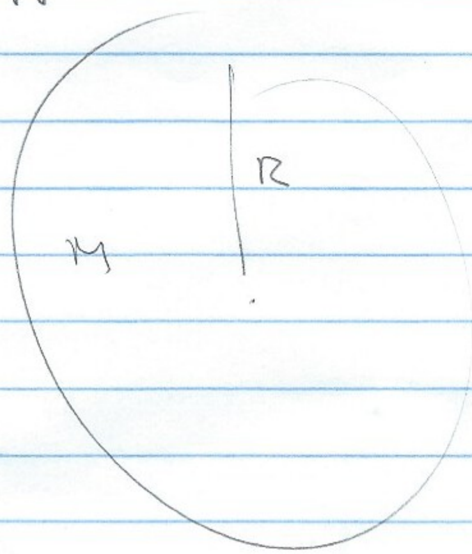

$R = .75\ m$

$M = 250\ kg$

$\omega_{max} \longleftrightarrow 1200\ rpm$

$I = \frac{1}{2} M R^2$

a) $I\alpha = \tau$ so $\alpha = \frac{\tau}{I}$, & $\omega_{max} = \alpha t_{max}$

So $t_{max} = \frac{\omega_{max}}{\alpha} = \frac{\omega_{max}}{\tau / I} = \frac{\omega_{max} I}{\tau}$

b.) $E = \frac{1}{2} I \omega_{max}^2$

c.) $\text{Power} = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}\left(\frac{1}{2} I \omega_{max}^2\right)}{2\text{ sec.}} = \frac{1}{8} \frac{I \omega_{max}^2}{\text{sec.}}$

d.) $E_{final} = \frac{1}{2} E_{max}$ so $\omega_{final} = \frac{1}{\sqrt{2}} \omega_{max}$

$\alpha_{dec} \Delta t = \Delta\omega = \frac{1}{\sqrt{2}} \omega_{max} - \omega_{max} = \left(\frac{1}{\sqrt{2}} - 1\right) \omega_{max}$

so $\alpha_{dec} = \frac{\left(\frac{1}{\sqrt{2}} - 1\right) \omega_{max}}{2\text{ sec}}$

$\tau_{dec} = I \alpha_{dec} = I\left(\frac{\left(\frac{1}{\sqrt{2}} - 1\right) \omega_{max}}{2\text{ sec}}\right)$

prob. 3) cont.

numerics:

$$\omega_{max} = \frac{1200\ (2\pi\ rad)}{60\ sec} = 40\pi\ \frac{rad}{sec}$$

$$I = \frac{1}{2}(250\ kg)\left(\frac{3}{4}m\right)^2 = \frac{9}{32}(250)\ kg\,m^2 = 70.31\ kg\,m^2$$

a) $$t_{max} = \frac{40\pi\ (70.31)}{50} = 176.71 \text{ seconds}$$

b) $$E = \frac{1}{2}(70.31)(40\pi)^2 \text{ joules} = .555 \times 10^6\ J$$

c) $$\frac{\Delta E}{\Delta t} = .1388 \times 10^6\ W$$

d) $$\frac{\left(\frac{1}{\sqrt{2}}-1\right)}{2}\ \frac{I\omega_{max}}{sec} = 1,293.92\ Nm \equiv \tau_{dec.}$$

$$1.29 \times 10^3\ Nm >> 5 \times 10^1\ Nm$$

$$\tau_{dec} >> \tau_{acc.}$$

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prob 4)

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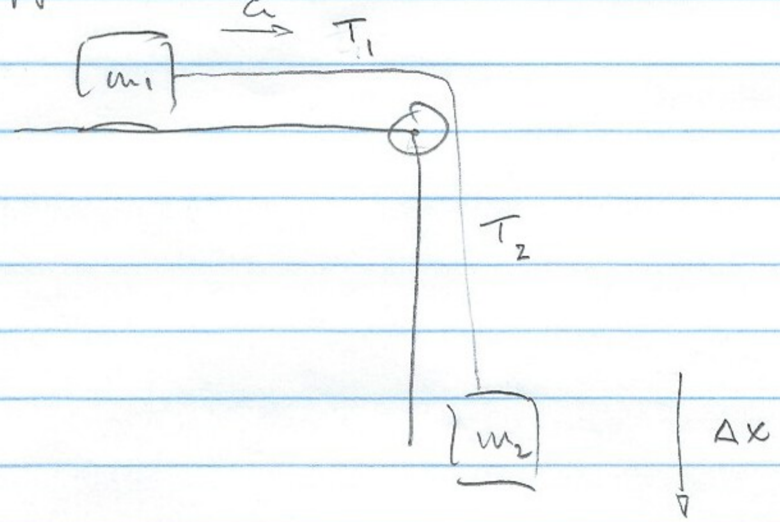

a) massless pulley: $(m_1 + m_2)\, a = m_2 g$

$$\text{so} \quad a = \frac{m_2}{(m_1 + m_2)}\, g$$

b) I use energy: ΔKE = work done <u>on</u> system

$$\frac{1}{2}(m_1 + m_2) v^2 + \frac{1}{2} I \omega^2 = m_2 g \Delta x$$

but $\omega R = v$ so $\omega = \frac{v}{R}$

$$\frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right) v^2 = m_2 g \Delta x$$

take derivative!

$$\left(m_1 + m_2 + \frac{I}{R^2}\right) a = m_2 g$$

$$\text{So} \quad a = \frac{m_2}{m_1 + m_2 + I/R^2}\, g$$

$T_1 = m_1 a$, & $m_2 g - T_2 = m_2 a$

$$T_1 = \left(\frac{m_1}{m_1 + m_2 + I/R^2}\right) m_2 g \qquad T_2 = \left(\frac{m_1 + I/R^2}{m_1 + m_2 + I/R^2}\right) m_2 g$$

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prob 5)

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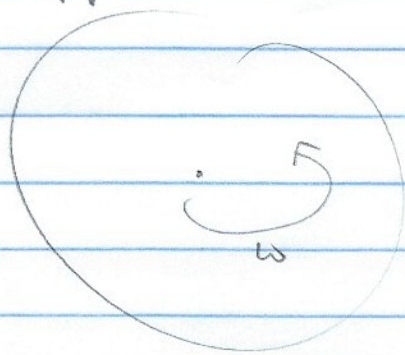
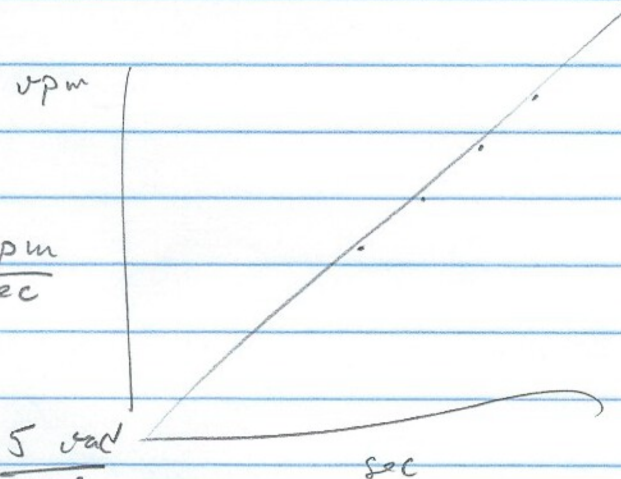


At constant frictional torque We expect

$\omega = \omega_0 - \alpha t$



When I graph ... I get a slope of $.7808 \times 10^2 \frac{\text{rpm}}{\text{sec}}$

$= .7808 \frac{(2\pi \text{ rad})}{(60 \text{ sec}) \text{ sec}} \times 10^2 = 8.1765 \frac{\text{rad}}{\text{sec}^2}$

If $\tau = I\alpha = (2.6 \text{ kgm}^2)(8.1765) \frac{\text{rad}}{\text{sec}^2}$

$\tau = 21.2589 \text{ Nm}$

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prob 6)

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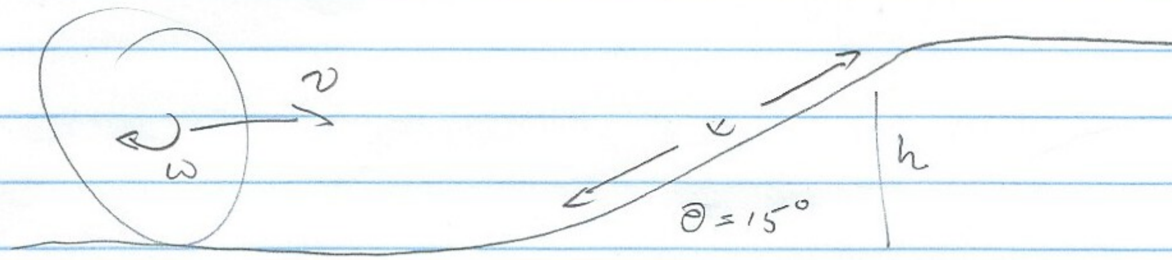

KE = PE

So $\frac{1}{2} I_{tot} \omega^2 = Mgh$

$\omega R = v$

$h = \frac{\frac{1}{2} I_{tot} v^2}{g M R^2}$

$\frac{h}{x} = \sin(\theta) = .2588$

$(I_{tot})_{disk} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

$(I_{tot})_{ring} = 1MR^2 + MR^2 = 2MR^2$

$h_{disk} = \frac{1}{2}\left(\frac{3}{2}\right)\frac{v^2}{g} = \frac{3}{4}\frac{(1.5)^2}{9.8} = .17$

$h_{ring} = \frac{1}{2}(2)\frac{v^2}{g} = \frac{(1.5)^2}{9.8} = .23$

$x_{disk} = \frac{h_d}{s\theta} = .67$ m

$x_{ring} = \frac{h_r}{s\theta} = .89$ m

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prob 7)

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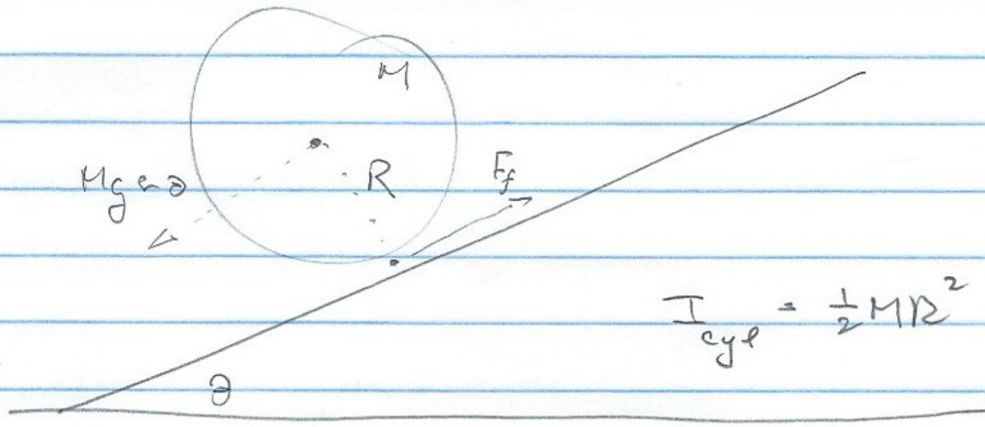

$I_{cyl} = \frac{1}{2}MR^2$

Great problem!

$$Mg\sin\theta - F_f = Ma$$

sub $\alpha R = a$

$$F_f R = I_0 \alpha$$

$$\frac{F_f R^2}{I} = a$$

so... $Mg\sin\theta - F_f = \frac{MR^2}{I} F_f$

$$\frac{Mg\sin\theta}{1 + MR^2/I} = F_f = \mu_s N = \mu_s Mg\cos\theta$$

$$\frac{Mg\sin\theta}{1+2} = \mu_s Mg\cos\theta$$

$$\frac{\tan\theta}{3} = \mu_s \quad \text{minimum required.}$$

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prob 8)

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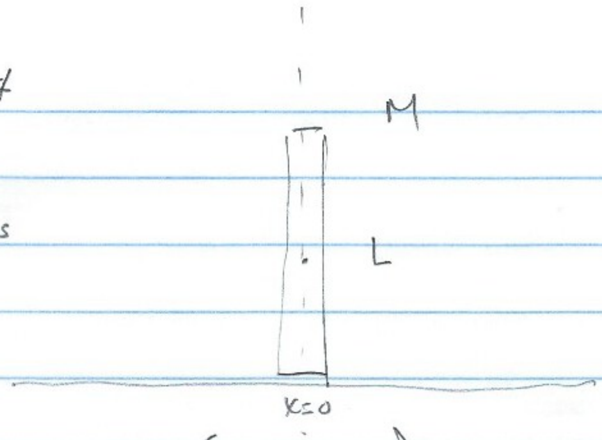
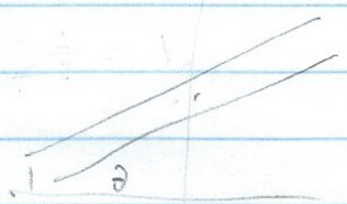
Two cases



$$\frac{1}{2} I_0 \omega_f^2 + \frac{1}{2} M v_{cm}^2 = Mg\frac{L}{2}$$



again $\omega_f \frac{L}{2} = v_{cm_f}$

$$\frac{1}{2}\left(\frac{1}{12}\cancel{M}L^{\cancel{2}}\right)\left(\frac{2v_{cm}}{\cancel{L}}\right)^2 + \frac{1}{2}\cancel{M} v_{cm}^2 = \cancel{M}g\frac{L}{2}$$

$$\left(\frac{1}{6} + \frac{1}{2}\right) v_{cm}^2 = g\frac{L}{2}$$

$$\frac{2}{3} v_{cm}^2 = g\frac{L}{2}$$

$$v_{cm}^2 = \frac{3}{4} gL$$

$$\frac{1}{2} I_{tot}\, \omega_f^2 = Mg\frac{L}{2}$$

$$\omega_f \frac{L}{2} = v_{cm_f}$$

$$\frac{1}{2}\left(\frac{1}{3}\cancel{M}L^{\cancel{2}}\right)\left(\frac{2v_{cm}}{\cancel{L}}\right)^2 = \cancel{M}g\frac{L}{2}$$

$$\frac{2}{3} v_{cm}^2 = g\frac{L}{2}$$

$$v_{cm}^2 = \frac{3}{4} gL$$

same!

book is wrong!

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prob 9)

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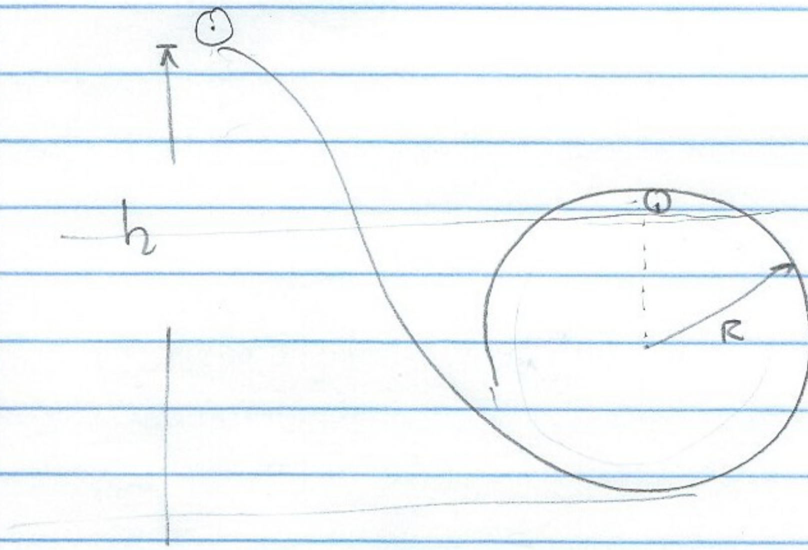

The marble is to make it around the loop-the-loop.
Its C.M. is traversing a circle of radius $R-r$
It must have a velocity greater than

$$\frac{v_{min}^2}{R-r} = g$$

The drop in height must be:

$$\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_0 \omega^2 = mg\left[(h+r) - (2R-r)\right]$$

$$\omega r = v_{cm} \quad \text{so} \quad I_0 \omega^2 = \frac{2}{5} m r^2 \omega^2 = \frac{2}{5} m v_{cm}^2$$

$$\frac{1}{2} \cancel{m} \left(v_{cm}^2 + \frac{2}{5} v_{cm}^2\right) = \cancel{m} g (h_{min} + 2r - 2R)$$

$$\frac{7}{5} v_{cm}^2 = 2g (h_{min} + 2r - 2R)$$

$$(R-r)\cancel{g} = \frac{10}{7}(h_{min} - 2(R-r)) \rightsquigarrow h_{min} = 2.7(R-r)$$

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prob 10)

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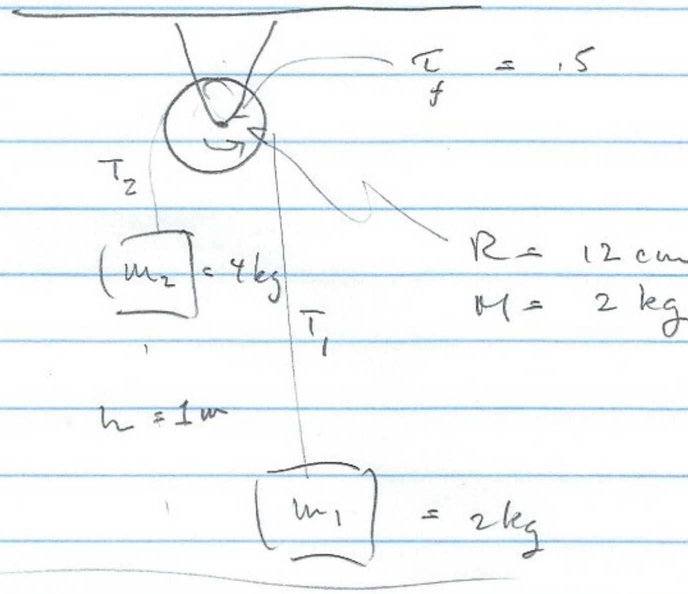

$$\begin{cases} m_1 a_1 = T_1 - m_1 g \\ m_2 a_2 = T_2 - m_2 g \end{cases} \qquad a_2 = -a_1$$

$$-\tau_f + (T_2 - T_1) R = I\alpha \qquad ; \quad \alpha R = a_1$$

Subtract $\quad (m_1 + m_2) a_1 = \underbrace{(T_1 - T_2)} - (m_1 - m_2) g$

$$I \alpha R = (T_2 - T_1) R^2 - \tau_f R$$

add

$$\frac{I}{R^2} a_1 = -(T_1 - T_2) - \tau_f / R$$

$$\left(m_1 + m_2 + \frac{I}{R^2}\right) a_1 = (m_2 - m_1) g - \tau_f / R$$

So $\quad a_1 = \dfrac{(m_2 - m_1) g - \tau_f / R}{m_1 + m_2 + I/R^2}$

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prob 10) continued

Now we know a_1

$$a_2 = -a_1$$

$$T_1 = m_1 a_1 + m_1 g$$

$$T_2 = -m_2 a_1 + m_2 g$$

The time to drop is:

$$\frac{1}{2} a_1 t^2 = h \quad \text{or} \quad t_{drop} = \sqrt{2h/a_1}$$