Introduction to the Finite Element Method

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Problem Statement

We want to find:

-Steady State Temperatures

-what matters, what doesn't?

Physical Model

Types of Heat Transfer:

Conduction

Convection
(Newton's Law of Cooling)

Radiation
(complicated, but small)

Physical Model

$$k\Delta u = f(x, y, z)$$

"Conduction"

"Convection"

Now what?

$$k\Delta u = f(x, y, z)$$

- "Guess a solution"?

- Separation of Variables & Fourier Series?

- Relaxation / Finite Difference?

- Finite Elements.

$$k\Delta u = f(x, y, z)$$ (Difficult)

↓

Finite Element Method

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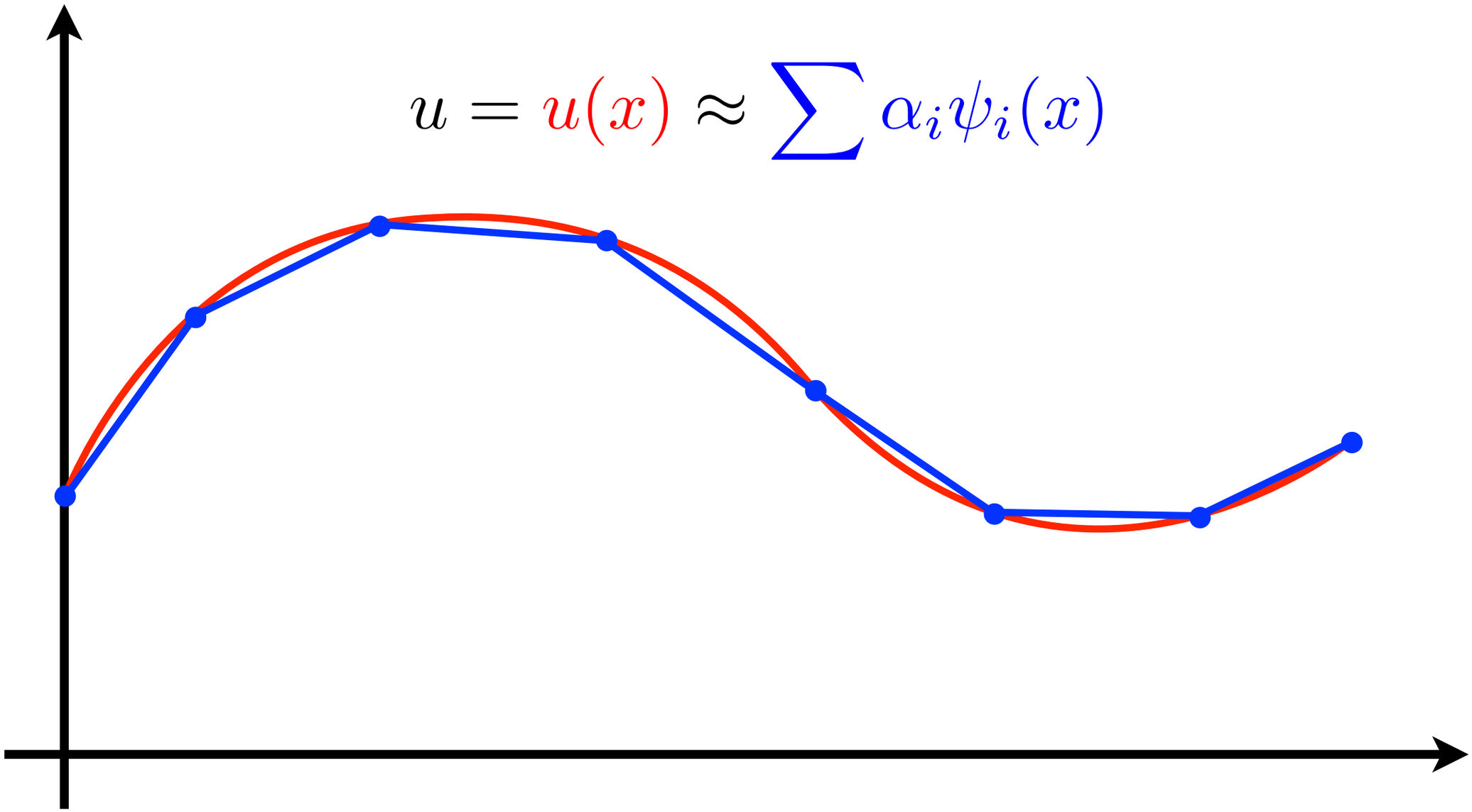
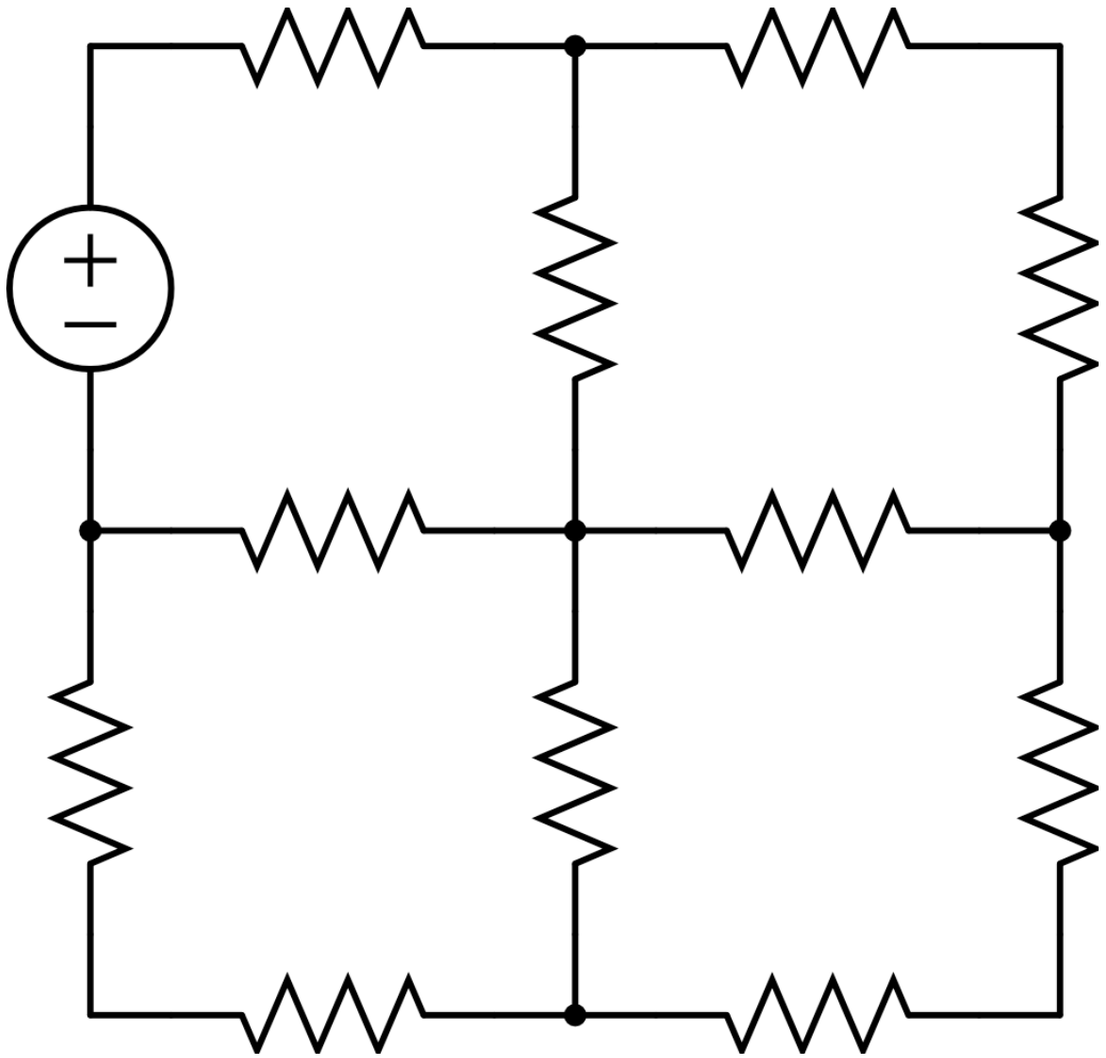
$$[K]\underline{u} = \underline{f}$$ (Easy)

Approximate Solutions

$$u = u(x) \approx \sum \alpha_i \psi_i(x)$$



Aside: Resistive Circuit



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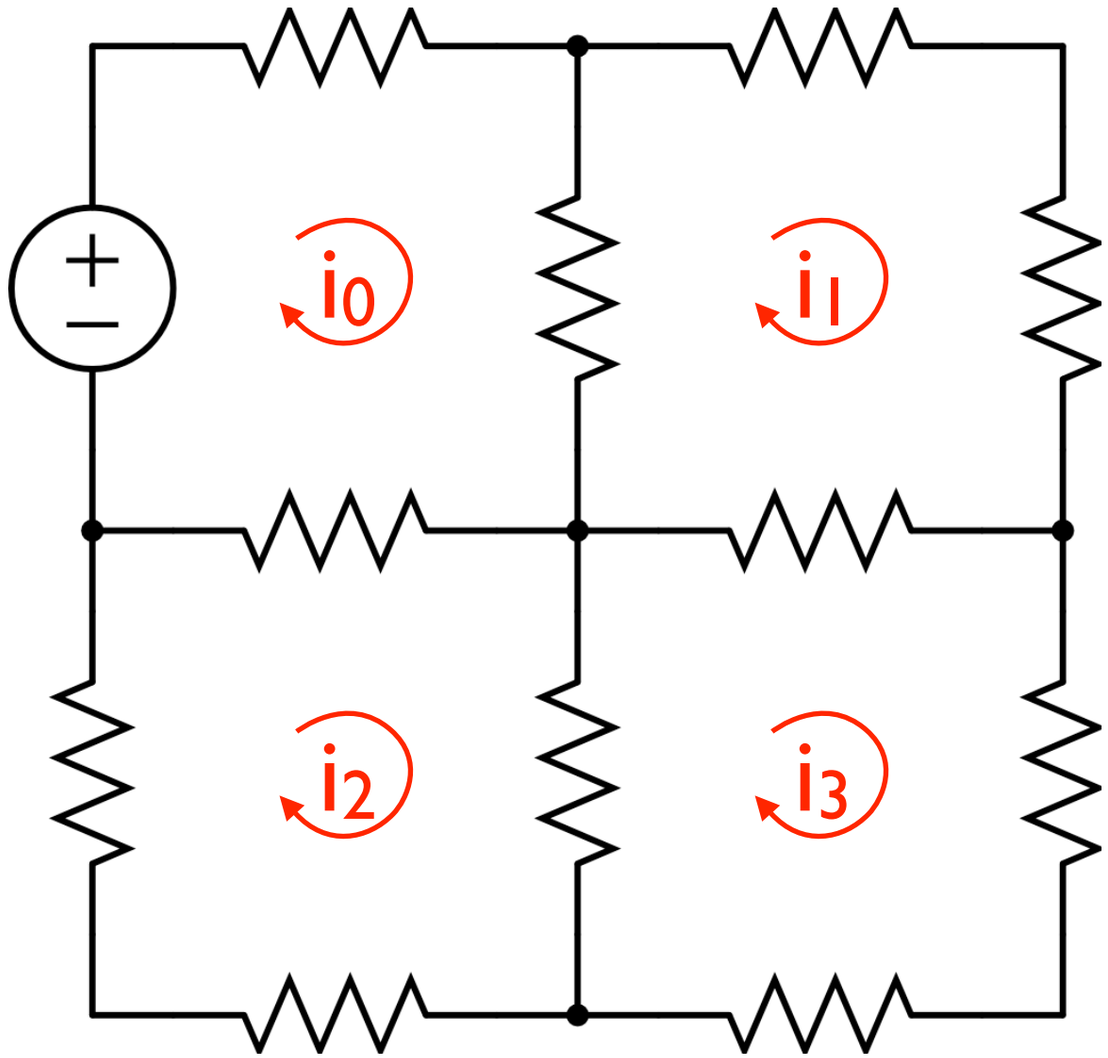
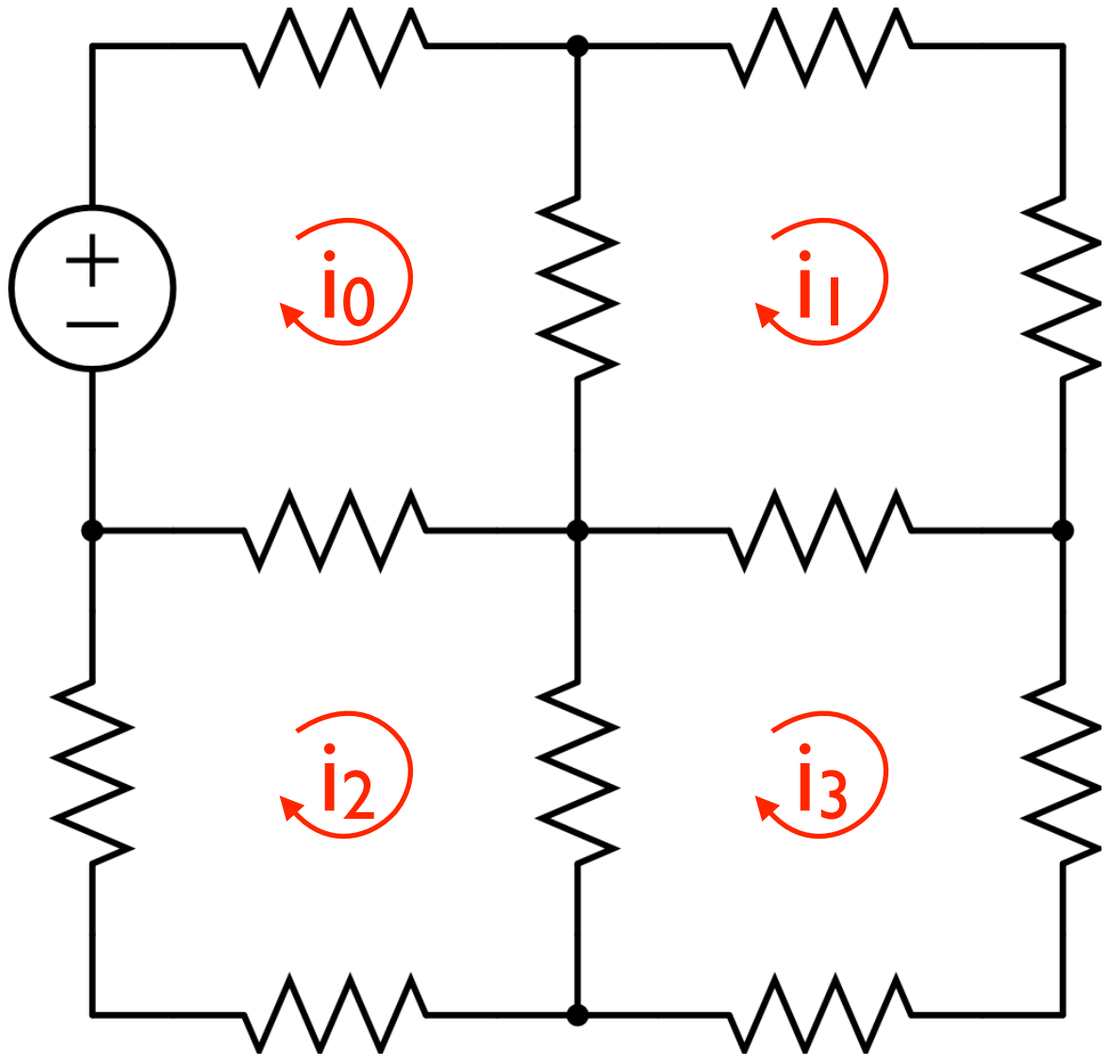


Mesh Analysis:

Aside: Resistive Circuit



Mesh Analysis:

$V = R(3i_0 - i_1 - i_2)$

$0 = R(4i_1 - i_0 - i_3)$

$0 = R(4i_2 - i_3 - i_0)$

$0 = R(4i_3 - i_1 - i_2)$

Aside: Resistive Circuit

-Easy

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

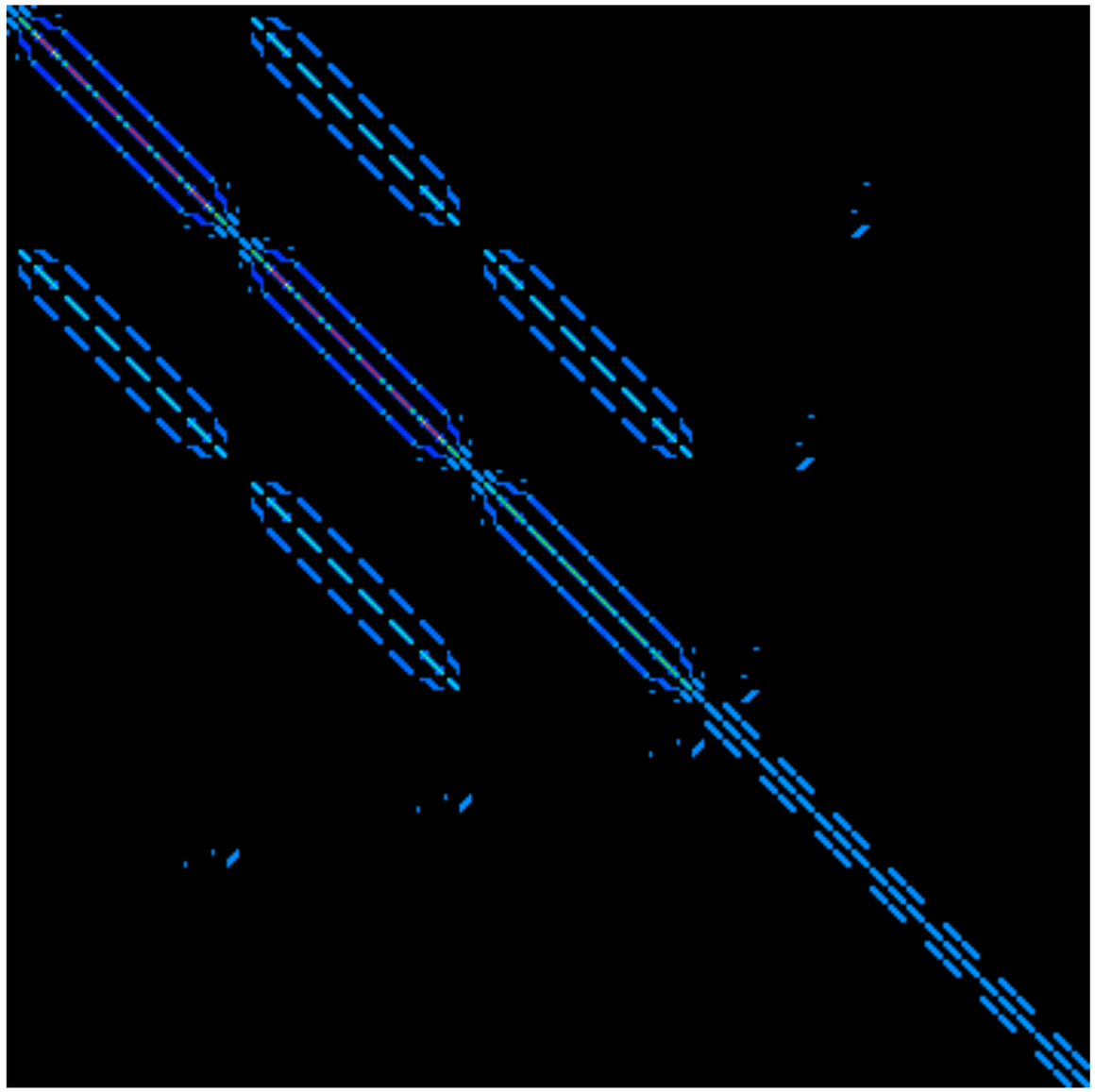
-Works for any resistive circuit

-Turned a PDE (difficult) into Linear Algebra (easy)

“Stiffness” Matrix Properties

-Sparsity structure

-Eigenvalues & Eigenvectors



Future Work

-Other applications

-Nonlinearities

-Fluids and Fracture

Thanks!

Questions?