# Describing Rotational Motion 

Pre-Class Questions

Problem Set (due next time)
Ch 9 -9, 14, 18, 23
Lecture Outline
I. The Definitions of the Rotational Variables
2. Rotational Kinematics

| Translational Variables | Rotational Variables |
| :--- | :--- |
| Position: <br> respect to a coordion of an object with <br> restem. | Angle: The rotational location of an object with respect <br> to a coordinate system. |
| Velocity: The rate of displacement. | $\underline{\text { Angular Velocity: The rate of angular displacement. }}$ |
| Acceleration: <br> velocity. | Angular Acceleration: <br> velocity..$~$ |

The units for angular quantities are often confusing. Let's work on some units associated with angles.

1. At the right, draw a bicycle wheel including the spokes.
2. Imagine one spoke rotating around. It makes an angle $\varnothing$ with its original position. Draw this for some angle $\varnothing$ less than $90^{\circ}$.
3. Now imagine the spoke completing exactly one rotation. Find the angle in revolutions, degrees, and radians.
4. Write the conversion factors for revolutions to degrees, degrees to radians, and radians to revolutions.
5. Convert $30^{\circ}$ to revolutions and radians.

$\qquad$ ${ }^{\circ}$

$30^{\circ}=$ $\qquad$ rev
$30^{\circ}=$ $\qquad$ rad
Now, let's think about the rate things spins.
6. At the right, draw a bicycle wheel including the spokes. Indicate that it is spinning.

7. Imagine one spoke rotating around. It completes one rotation in a time $T$ which could be in minutes or seconds. Write it's angular speed in degrees per time, revolutions per timev and radians per time.
$\omega=$ $\qquad$ $\left({ }^{\circ} / \mathrm{m}\right)$
$\omega=$ $\qquad$ (rev/m)
$\omega=$ $\qquad$ (rad/s)
8. Convert one revolution per minute to radians per second.


| Translational Variables | Rotational Variables | Relationship |
| :--- | :--- | :--- |
| Position: x | Angle: $\theta$ | $\mathrm{s}=\mathrm{r} \theta$ |
| Velocity: $v \equiv \frac{\Delta x}{\Delta t}$ | Angular Velocity: $\omega \equiv \frac{\Delta \theta}{\Delta t}$ | $v \equiv \frac{\Delta s}{\Delta t}=\frac{r \Delta \theta}{\Delta t} \Rightarrow v_{t}=r \omega$ |
| Acceleration: $a \equiv \frac{\Delta v}{\Delta t}$ | Angular Acceleration: $\alpha \equiv \frac{\Delta \omega}{\Delta t}$ | $a_{c}=\frac{v_{t}^{2}}{r}=\frac{(r \omega)^{2}}{r} \Rightarrow a_{c}=\omega^{2} r$ |

Example I: A CD 12.0cm in diameter is placed in a drive. It starts at rest and reaches 200rpm in I.20s. Find (a)the average angular acceleration and (b)the radial acceleration of a point on the rim when the rotation rate is 200rpm.

Below is a sketch of a merry go round. Rank these horses by their angular speed.


Below is a sketch of a merry go round. Rank these horses by their tangential speed.


## The Kinematic Equations

$$
\begin{aligned}
& v \equiv \frac{\Delta x}{\Delta t} \\
& \left.\begin{array}{l}
a \equiv \frac{\Delta v}{\Delta t} \\
\Delta a=0
\end{array}\right\} \text { a bunch of math } \Rightarrow\left\{\begin{array}{c}
v=v_{o}+a t \\
x=x_{o}+v_{o} t+\frac{1}{2} a t^{2} \\
v^{2}=v_{o}^{2}+2 a\left(x-x_{o}\right)
\end{array}\right. \\
& \left.\begin{array}{l}
\omega \equiv \frac{\Delta \theta}{\Delta t} \\
\alpha \equiv \frac{\Delta \omega}{\Delta t} \\
\Delta \alpha=0
\end{array}\right\} \text { the very same math } \Rightarrow\left\{\begin{array}{c}
\omega=\omega_{o}+\alpha t \\
\theta=\theta_{o}+\omega_{o} t+\frac{1}{2} \alpha t^{2} \\
\omega^{2}=\omega_{o}^{2}+2 \alpha\left(\theta-\theta_{o}\right)
\end{array}\right.
\end{aligned}
$$

Example 2: For the CD of example I, find (a)the number of revolutions and (b)the distance traveled by a point on the edge of the CD.

## Lecture 22 - Summary

The Definitions of Rotational Variables

| Translational Variables | Rotational Variables | Relationship |
| :--- | :--- | :--- |
| Position: x | Angle: $\theta$ | $\mathrm{s}=\mathrm{r} \theta$ |
| Velocity: $v \equiv \frac{\Delta x}{\Delta t}$ | $\underline{\text { Angular Velocity: } \omega \equiv \frac{\Delta \theta}{\Delta t}}$ | $v \equiv \frac{\Delta s}{\Delta t}=\frac{r \Delta \theta}{\Delta t} \Rightarrow v_{t}=r \omega$ |
| $\underline{\text { Acceleration: } a \equiv \frac{\Delta v}{\Delta t}}$ | Angular Acceleration: $\alpha \equiv \frac{\Delta \omega}{\Delta t}$ | $a_{c}=\frac{v_{t}^{2}}{r}=\frac{(r \omega)^{2}}{r} \Rightarrow a_{c}=\omega^{2} r$ |

If the angular acceleration is constant the kinematic equations apply.

