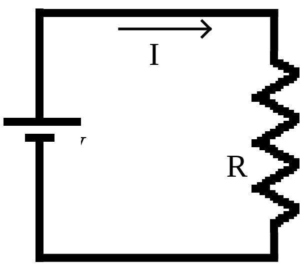
Chapter 28 - Circuits

Problem Set #7 - due:
Ch 28 - 1, 9, 14, 17, 23, 38, 47, 53, 57, 66, 70, 75

Lecture Outline
1. Kirchoff's Rules
2. Resistors in Series
3. Resistors in Parallel
4. More Complex Circuits
5. Electrical Meters
6. RC Circuits



In a circuit, charges move from one place to another carrying energy. These charges can be thought of as buckets that carry energy around a circuit. The battery fills the buckets. The buckets are emptied at various places around the circuit, but the buckets themselves never disappear. They return to the battery to be refilled. These basic ideas are summarized in Kirchoff's Rules and are applicable to even the most complicated circuits.

1. Kirchoff's Rules

The Junction Theorem:

"The current into any junction is exactly equal to the current out of the junction."

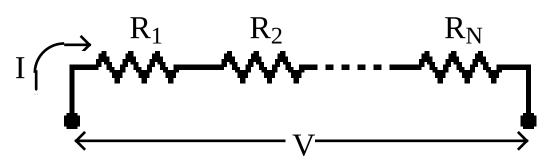
This theorem is explained by the Law of Conservation of Charge.

The Loop Theorem:

"The sum of all the voltage drops around any loop in a circuit must be zero."

This theorem is explained by The Law of Conservation of Energy and the fact that the electric force is conservative.

2. Resistors in Series



The loop theorem requires: $V - V_1 - V_2 - \cdots - V_N = 0 \qquad V = V_1 + V_2 + \cdots + V_N$

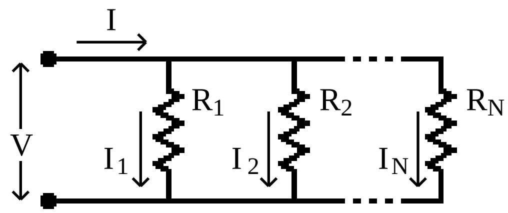
Ohm's Law says: $V = IR,\ V_1 = I_1R_1,\ V_2 = I_2R_2,\ \cdots V_N = I_NR_N \qquad IR = I_1R_1 + I_2R_2 + \cdots + I_NR_N$

The junction theorem means that: $I = I_1 = I_2 = \cdots = I_N \qquad IR = IR_1 + IR_2 + \cdots + IR_N$

$$R = R_1 + R_2 + \cdots + R_N$$

$$\boxed{R_s = \sum R_i \quad \text{Resistors in Series}}$$

3. Resistors in Parallel



The junction theorem means that: $I = I_1 + I_2 + \cdots + I_N$

Ohm's Law says: $I = \dfrac{V}{R},\ I_1 = \dfrac{V_1}{R_1},\ I_2 = \dfrac{V_2}{R_2},\ \cdots I_N = \dfrac{V_N}{R_N} \qquad \dfrac{V}{R} = \dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \cdots + \dfrac{V_N}{R_N}$

The loop theorem requires: $V - V_1 = 0,\ V - V_2 = 0,\ \cdots V - V_N = 0 \qquad V = V_1 = V_2 = \cdots = V_N$

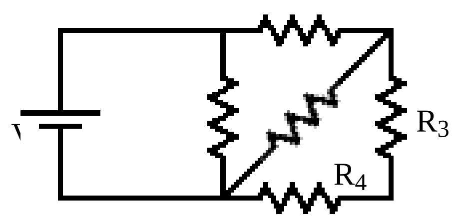
So, $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2} + \cdots + \dfrac{V}{R_N} \qquad \dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_N}$

$$\boxed{\frac{1}{R_p} = \sum \frac{1}{R_i} \quad \text{Resistors in Parallel}}$$

4. More Complex Circuits

1) Circuit elements in series have the same current, but divide up a common voltage.
2) Circuit elements in parallel have the same voltage, but divide up a common current.

Many resistor circuits are just combinations of series and parallel. They can be studied using the series and parallel rules. This is called "circuit reduction." Other circuits are not combinations of series and parallel. These circuits have to be examined with Kirchoff's Rules.



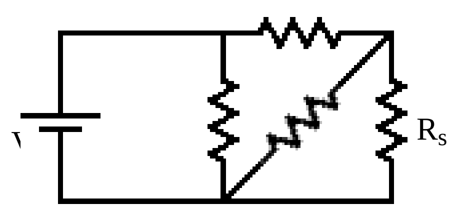
Example 1: For the circuit shown find (a)the equivalent resistance, (b)the current provided by the battery and (c)the current through and voltage across each resistor (V=90.0V, R_1=3.00kΩ, R_2=4.00kΩ, R_3=1.00kΩ, R_4=2.00kΩ and R_5=6.00kΩ).

(a)Use the idea of circuit reduction.

R_3 and R_4 are in series.

They can be replaced with an equivalent resistor,

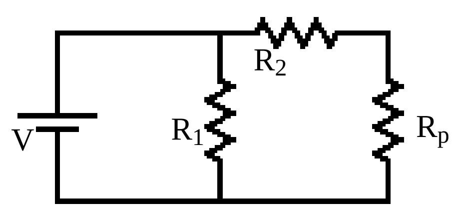
$R_s = R_3 + R_4 = 3.00\text{k}\Omega$.

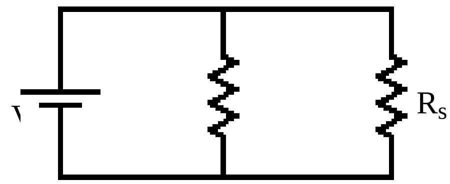


R_s and R_5 are in parallel.

They can be replaced with an equivalent resistor,

$\dfrac{1}{R_p} = \dfrac{1}{R_s} + \dfrac{1}{R_5} \qquad R_p = 2.00\text{k}\Omega$.





R_p and R_2 are in series.

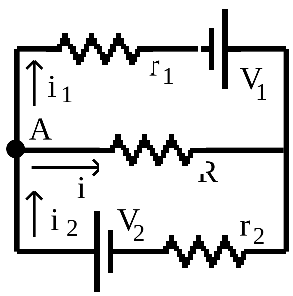
They can be replaced with an equivalent resistor, $R_s = R_p + R_2 = 6.00k\Omega$.

R_s and R_1 are in parallel. The equivalent resistor is, $\frac{1}{R} = \frac{1}{R_s} + \frac{1}{R_1}$ $\quad R = 2.00k\Omega$.

(b)Use Ohm's Rule $V = IR$ $\quad I = \frac{V}{R} = \frac{90.0V}{2.00k\Omega} = \underline{\underline{45.0mA}}$

(c)Follow the current and voltage drops around the circuit using Kirchoff's Rules and Ohm's Rule. In summary,

V(V)	I(mA)	R(kΩ)
90.0	30.0	3.00
60.0	15.0	4.00
10.0	10.0	1.00
20.0	10.0	2.00
30.0	5.00	6.00



<u>Example 2:</u> *A 6.00V battery (V_1) with a 2.00Ω internal resistance and a 3.00V battery with a 3.00Ω internal resistance is connected to a 6.00Ω resistor as shown. Find the terminal voltage for each battery and the current through the 6.00Ω resistor.*

This circuit is not a combination of series and parallel resistor, so we must go back to the more basic ideas of Kirchoff's Rules to solve the problem. Applying the junction theorem at point A $\quad i_2 = i + i_1$. The loop theorem around the lower loop $\quad V_2 - iR - i_2 r_2 = 0$ and around the upper loop $\quad V_1 + iR - i_1 r_1 = 0$.

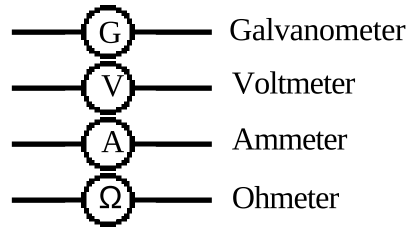
This gives three equations for the three unknown currents. Solving the loop theorem equations for i_1 and i_2,

$i_2 = \frac{V_2 - iR}{r_2}$ and $i_1 = \frac{V_1 + iR}{r_1}$. Substituting into the junction theorem equation, $\frac{V_2 - iR}{r_2} = i + \frac{V_1 + iR}{r_1}$.

Solving for i, $i = \frac{V_2 - V_1\frac{r_2}{r_1}}{R + r_2 + R\frac{r_2}{r_1}} = \underline{\underline{-0.333A}}$ the minus sign means that we chose the direction of this current wrong. Substituting back for i_1 and i_2, $i_1 = \underline{\underline{2.00A}}$ and $i_2 = \underline{\underline{1.67A}}$.

The terminal voltage is the actual potential difference across the terminals of the battery, $V_{t1} = V_1 - i_1 r_1 = \underline{\underline{2.00V}}$ and $V_{t2} = V_2 - i_2 r_2 = \underline{\underline{-2.00V}}$

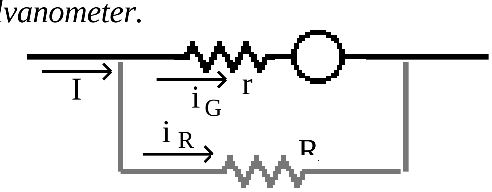
5. Electrical Meters



The basic constituent part of a voltmeter, ammeter, or ohmmeter is a galvanometer. Mechanical galvanometers are made from a coil of wire and a magnet. Modern galvanometers are IC chips. They all function in the same basic way. They respond linearly to small currents. The different types of meters are constructed from a galvanometer and properly placed resistors.

The schematic symbols for different types of meters are shown at the right.

<u>Example 3:</u> *A galvanometer of internal resistance 0.120Ω reads full scale when 15.0μA passes through it. Design an ammeter to read up to 1.00A with this galvanometer.*

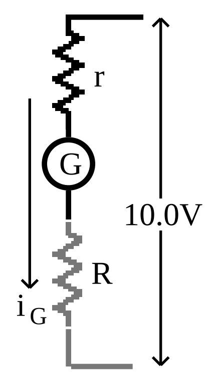


If the entire 1.00A goes through the galvanometer it will blow up. We must provide an alternate path for the current. We can control the fraction of the current that goes through the galvanometer with a resistor called a "shunt."

Using the junction theorem, $I = i_G + i_R$. The loop theorem requires $i_G r - i_R R = 0$. Solving for i_R and substituting into the junction theorem equation ,

$$i_R = \frac{r}{R} i_G \qquad I = i_G\left(1 + \frac{r}{R}\right) \qquad R = \frac{r}{\left(\frac{I}{i_G}\right) - 1} = \underline{\underline{1.80\mu\Omega}}.$$

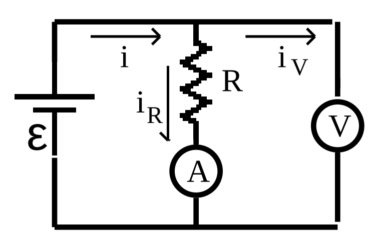
<u>Example 4:</u> *Use an identical galvanometer to build a voltmeter to measure up to 10.0V.*



If the entire 10.0V is across the galvanometer the current will be huge and it will cook. We must cut down the current that goes through the galvanometer with a resistor in series. Using the loop theorem, $V = i_G r + i_G R$.

Solving for R, $R = \frac{V}{i_G} - r = \underline{\underline{6.67\text{x}10^5\,\Omega}}$.

<u>Example 5:</u> *The ammeter from example 3 and the voltmeter from example 4 are used to measure the resistance in the circuit shown. Find the difference between the ratio of the meter readings and the true resistance.*



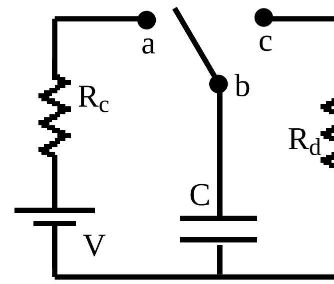
Using the loop theorem, $V = i_R(R + R_A)$ where V is the voltmeter reading and R_A is the resistance of the ammeter which can be found from the parallel rule,

$$\frac{1}{R_A} = \frac{1}{r} + \frac{1}{R_{shunt}} \qquad R_A = \frac{rR_{shunt}}{r + R_{shunt}} \approx R_{shunt} = 1.80\mu\Omega.$$

Using $R' \equiv \frac{V}{i_R}$ as the ratio of the meter readings the equation from the loop theorem can be written,

$i_R R' = i_R(R + R_A) \qquad R' = R + R_A = R + R_{shunt}$. The difference between the meter ratio and the actual resistance is, $R' - R = R_{shunt} = \underline{\underline{1.80\mu\Omega}}$.

6. RC Circuits



Kirchoff's Rules are very widely applicable. This is not surprising considering they come from the Laws of Conservation of Energy and Conservation of Charge. They can be used to analyze circuits with both resistors and capacitors.

A typical RC circuit is shown at the right. When the switch connects a and b current flows and the battery begins to charge the capacitor. When the capacitor is charged current can no longer flow. The question is, how long does this take?

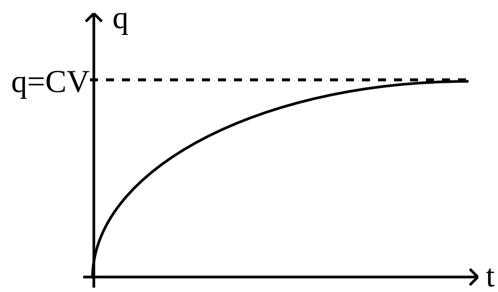
At some intermediate time the current in the circuit is i and the charge on the capacitor is q. Applying the loop theorem, $V - iR_c - \frac{q}{C} = 0$. From the definition of current, the current must equal the rate at which the capacitor charges, $i = \frac{dq}{dt}$. The equation from the loop theorem becomes, $V - \frac{dq}{dt}R_c - \frac{q}{C} = 0$. This equation can be solved for q(t) by solving for $\frac{dq}{dt}$ and integrating,

$$\frac{dq}{dt} = \frac{CV - q}{R_cC} \qquad \frac{dq}{CV - q} = \frac{dt}{R_cC} \qquad \int_0^q \frac{dq}{CV - q} = \int_0^t \frac{dt}{R_cC} \qquad -\int_{CV}^{CV-q} \frac{du}{u} = \frac{t}{R_cC} \qquad \ln\left(\frac{CV - q}{CV}\right) = -\frac{t}{R_cC}$$

Solving for q,

$$\boxed{q = CV\left(1 - e^{-t/RC}\right) \quad \text{Charging RC Circuit}}$$

The graph of the charge on the capacitor as a function of time is shown.



Note:
1) The charge is initially zero.
2) The charge grows exponentially.
3) The maximum charge occurs when the capacitor voltage matches the battery voltage.

When the switch connects b and c the capacitor discharges. The loop theorem requires, $-iR_d + \frac{q}{C} = 0$.

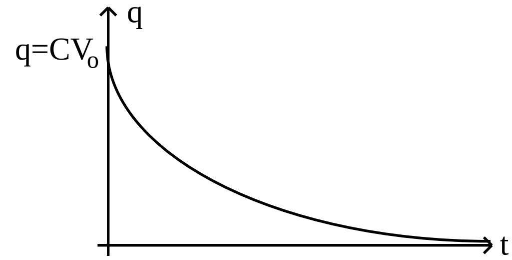
The current must equal the rate at which the capacitor discharges, $i = -\frac{dq}{dt}$.

The equation from the loop theorem becomes, $\frac{dq}{dt} = -\frac{q}{R_dC}$.

Integrating, $\frac{dq}{q} = -\frac{dt}{R_dC} \qquad \int_{CV_o}^q \frac{dq}{q} = -\int_0^t \frac{dt}{R_dC} \qquad \ln\left(\frac{q}{CV_o}\right) = -\frac{t}{R_dC} \qquad q = CV_o e^{-t/R_dC}$

$$\boxed{q = CV_o e^{-t/RC} \quad \text{Discharging RC Circuit}}$$

The graph of the charge on the capacitor as a function of time is shown.



Note:

1)The charge is initially the capacitance times the initial voltage on the capacitor.
2)The charge dies exponentially.

<u>Example 6:</u> A 5.00μF capacitor is charged to 10.0V. A 10.0cm piece of 2.00mm diameter copper wire is used to short it out. Find the time it takes for the capacitor's voltage to drop to 10.0mV.

For the discharge of a capacitor, $q = CV_o e^{-t/RC}$ $\quad \frac{q}{C} = V_o e^{-t/RC}$ $\quad V = V_o e^{-t/RC}$.

Solving for the time, $\frac{V}{V_o} = e^{-t/RC}$ $\quad \ln\left(\frac{V}{V_o}\right) = -\frac{t}{RC}$ $\quad t = -RC\ln\left(\frac{V}{V_o}\right)$.

The resistance can be found from its definition,

$$R = \rho\frac{\ell}{A} = (1.7x10^{-8})\frac{0.100}{\pi(0.00100)^2} = 5.41x10^{-4}\,\Omega.$$

Putting the numbers in,

$$t = -RC\ln\left(\frac{V}{V_o}\right) = (5.41x10^{-4})(5.00x10^{-6})\ln\left(\frac{0.0100}{10.0}\right) = 1.9x10^{-8}\text{s} = \underline{\underline{19\text{ns}}}.$$

<u>Chapter 28 - Summary</u>

Kirchoff's Rules:

The Junction Theorem: "The current into any junction is exactly equal to the current out of the junction."

The Loop Theorem: "The sum of all the voltage drops around any loop in a circuit must be zero."

Resistors in Series $R_s = \sum R_i$

Resistors in Parallel $\frac{1}{R_p} = \sum \frac{1}{R_i}$

Charging RC Circuit $q = CV\left(1 - e^{-t/RC}\right)$

Discharging RC Circuit $q = CV_o e^{-t/RC}$