Chapter 32 - Magnetism of Matter; Maxwell's Equations

Problem Set #11 - due:
Ch 32 - 3, 7, 11, 18, 20, 21, 22, 30, 31, 34, 38, 45

Lecture Outline
1. Magnets as Dipoles
2. Gauss' Law for Magnetism
3. Magnetism in Matter
4. Three Types of Magnetic Behavior
5. Induced Magnetic Fields
6. Maxwell's Equations

At this point we know the laws that describe electric fields. They are,

Gauss's Law for Electricity $\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_o}$ [Charge creates diverging E fields]

Faraday's Law of Induction $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ [Changing B's create circulating E's]

The laws that explain the properties of magnetic fields aren't complete. As far as we know, the only way to create a magnetic field is with a current and this relationship is given by,

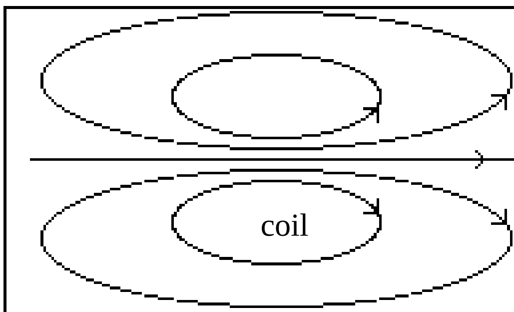
Ampere's Law $\oint \vec{B}\cdot d\vec{s} = \mu_o \frac{dq}{dt}$ [Currents make circulating B fields]

In this chapter we will complete the laws of magnetism by examining magnets and by finding a way to induce magnetic fields. The complete laws of electricity and magnetism are known as "Maxwell's Equations."

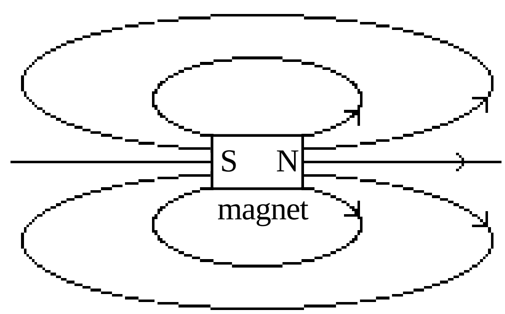
1. Magnets as Dipoles

By looking at the laws for electric fields you begin to wonder about $\oint \vec{B}\cdot d\vec{A} = ?$.

In other words, are there ways to make diverging magnetic fields. Is there something analogous to electric charges that produce diverging magnetic fields. The obvious guess would be a magnet.



The magnetic field lines due to a coil.



The magnetic field lines due to a magnet.

Magnetic Filing Demo w/ wire, coil, magnet and power supply

The magnetic field of a magnet isn't diverging. A magnet produces a field that is the same shape as the field due to a coil. Therefor a magnet is a dipole. That is why breaking a magnet in half doesn't separate the north pole from the south pole. Instead it creates two new magnets.

There is no known way to create a single magnetic pole.

2. Gauss' Law for Magnetism

Beyond the Mechanical Universe (vol. 34 Ch 18,19,20,21,22)

Gauss's Law for electric fields states that the electric flux through a closed surface is proportional to the enclosed charges. The same statement can be made for the magnetic flux, $\oint \vec{B} \cdot d\vec{A} = \mu_o q^{(m)}_{enclosed}$ where $q^{(m)}_{enclosed}$ is the magnetic charge of a "magnetic monopole." Since magnetism is always caused by currents there are only magnetic dipoles, therefor, $q^{(m)}_{enclosed} = 0$ and Gauss's Law for Magnetic Fields is,

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss's Law for Magnetism}$$

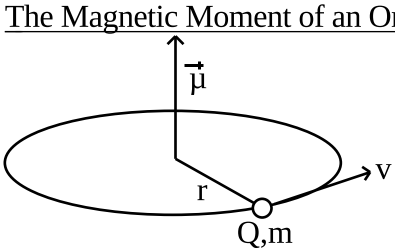
Theories of the interaction of fundamental particles often predict the existence of magnetic monopoles. These theories are always suspect because no magnetic monopole has ever been found.

3. Magnetism in Matter

Beyond the Mechanical Universe (vol. 35 Ch 14,15,16)

The macroscopic properties of matter are a manifestation of the microscopic properties of the atoms of which it is composed. The magnetic dipole moments of moving electrons, protons, and neutrons create the magnetic fields of bulk materials. The motions of these particles can be broken down into orbital motion (e.g. electrons) and spinning motion.

The Magnetic Moment of an Orbiting Charge



The current created by the orbiting charge is, $I \equiv \frac{dQ}{dt} = \frac{Q}{2\pi r/v} = \frac{Qv}{2\pi r}$.

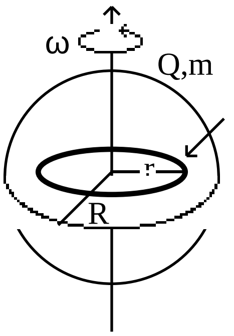
The magnetic moment of the orbiting charge is

$\vec{\mu} \equiv I\vec{A} \qquad \mu_o = \frac{Qv}{2\pi r}\pi r^2 = \frac{1}{2}Qvr = \frac{Q}{2m}\ mvr = \frac{Q}{2m}\ L_o$.

Putting in the vector signs, $\vec{\mu}_o = \frac{Q}{2m}\ \vec{L}_o$

> *Example 1: An electron in the ground state of the hydrogen atom has an orbital angular momentum of 1.05x10^{-34}J·s. Find the orbital magnetic moment.*
>
> The magnetic moment due to an orbiting charge is
>
> $\mu_o = \frac{Q}{2m}\ L_o = \frac{1.60\text{x}10^{-19}}{2(9.11\text{x}10^{-31})}(1.05\text{x}10^{-34}) = \underline{\underline{9.22\text{x}10^{-24}\text{A m}^2}}$.
>
> This is called the "Bohr Magneton" and it is verified experimentally.

The Magnetic Moment of a Spinning Charge



The current created by a ring of spinning charge, dq, is,

$$dI \equiv \frac{dq}{T} = \frac{dq}{2\pi/\omega} = \frac{\omega dq}{2\pi}.$$

The magnetic moment of the spinning ring of charge is

$$d\mu_s \equiv dIA \qquad d\mu_s = \frac{\omega dq}{2\pi}\pi r^2 = \tfrac{1}{2}\omega r^2 dq.$$

Summing over the rings, $\mu_s = \frac{1}{2}\omega\int r^2 dq$.

For lack of a better idea assume that the charge is distributed in the same way as the mass, $\frac{dq}{Q} = \frac{dm}{m}$ $\quad \mu_s = \frac{Q}{2m}\omega\int r^2 dm$. The integral is the rotational inertia and the product of the rotational inertia and the angular speed is the angular momentum. Now the magnetic moment can be written, $\vec{\mu}_s = \frac{Q}{2m}\vec{L}_s$ which is just like the result for the orbiting charge.

Example 2: An electron is known to have a spin angular momentum of $0.527\text{x}10^{-34}$ J·s. Find the spin magnetic moment.

The magnetic moment due to a spinning charge is

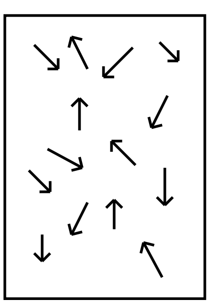
$$\mu_s = \frac{Q}{2m}L_s = \frac{1.60\text{x}10^{-19}}{2(9.11\text{x}10^{-31})}(0.527\text{x}10^{-34}) = \underline{\underline{4.63\text{x}10^{-24}\text{A m}^2}}.$$

The bad news is that experiments show that the actual number is twice as big as this result. The problem lies with the assumption about the charge distribution. To correct for this mistake the spin magnetic moment is usually written, $\vec{\mu}_s = g\frac{Q}{2m}\vec{L}_s$ where g is called the "gyromagnetic ratio." Its value is g=2 for an electron.

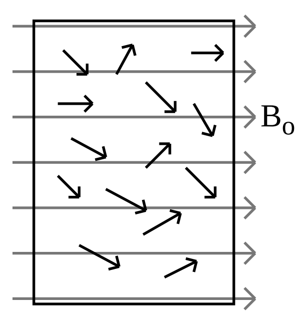
In summary, the magnetic moment of a moving charge is directly proportional to its angular momentum.

$$\vec{\mu} = \frac{q}{2m}\vec{L} \quad \text{The Magnetic Moment of a Charge}$$

Magnetization



In a bulk material where no applied magnetic field is present the magnetic dipoles are randomly aligned as shown at the left. The total magnetic field due to all the dipoles cancels to zero. When a field, B_o, is applied to the material, the dipoles tend to align with the applied field. In turn, they now produce a net field, B_a. This field due to the aligned dipoles can, in principle, be calculated by adding up the fields of the individual dipoles.



The total field due to alignment should be, $\vec{B}_a = \Sigma\vec{B}_i$.

The field due to an individual dipole roughly the field on the axis of a very small ring,

$$\vec{B}_i = \frac{\mu_o I}{2}\,\frac{a^2}{\left(a^2+r_i^2\right)^{3/2}}\,\hat{r}_i .$$

Since $r_i >> a$, $\vec{B}_i \approx \frac{\mu_o I}{2}\,\frac{a^2}{r_i^3}\hat{r}_i = \frac{\mu_o I}{2\pi}\,\frac{\pi a^2}{r_i^3}\hat{r}_i = \frac{\mu_o}{2\pi}\,\frac{\vec{\mu}_i}{r_i^3}$.

The field due the alignment is, $\vec{B}_a = \Sigma\frac{\mu_o}{2\pi}\,\frac{\vec{\mu}_i}{r_i^3} \approx \mu_o\frac{\vec{\mu}}{vol}$ where $\frac{\vec{\mu}}{vol}$ is some sort of average dipole moment per unit volume. This quantity is defined to be the "Magnetization."

$\vec{M} \equiv \frac{\vec{\mu}}{vol}$	The Definition of Magnetization

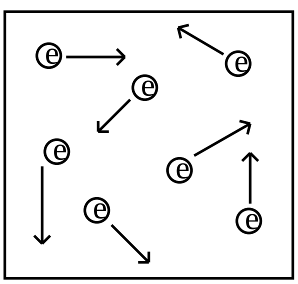
The field due to the aligned dipoles can be written as, $\vec{B}_a = \mu_o\vec{M}$

The total magnetic field is $\vec{B} = \vec{B}_o + \mu_o\vec{M}$.

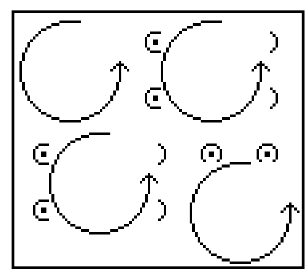
4. Three Types of Magnetic Behavior

Paramagnetism: This is the most common form of magnetic behavior. The spin magnetic moments tend to align with an applied magnetic field. Typically complete alignment is prevented by the thermal motions of the atoms.

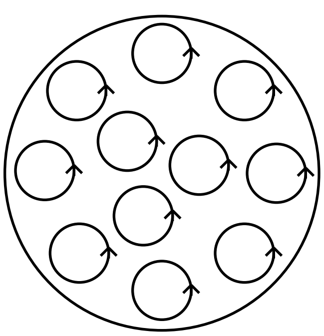
paramagnetism: $M > 0$ but fairly small.



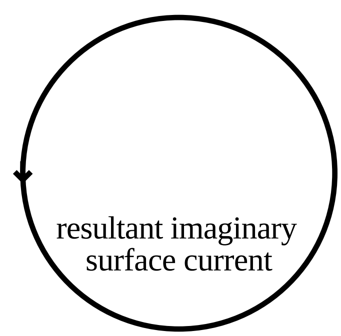
Diamagnetism: Materials that have very small spin magnetic moments can actually produce a magnetic field that is opposite to the applied field. This is caused by the random motion of electrons. In the absence of the applied field the electrons move randomly as shown on the left. When the field is applied out of the page as shown at the right, they begin to move in clockwise circular orbits as shown. These orbiting electrons create a field into the page, opposite to the applied field.

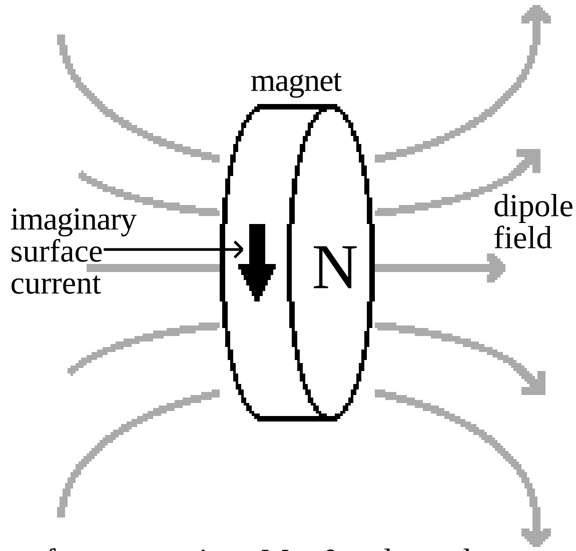


diamagnetism: $M < 0$ but very small.



Ferromagnetism: This form of magnetic behavior explains the common magnet. The spin magnetic moments in these materials are extremely easy to align. So easy, in fact, they can keep each other aligned. The spin magnetic moments of the atoms are shown at the left. The currents due to these moments tend to cancel except at the surface. The same net effect can be created by replacing the individual currents with a single imaginary current around the outer surface. The resulting field can be thought of as if it were caused by this current. The field due to such a magnet is shown below.





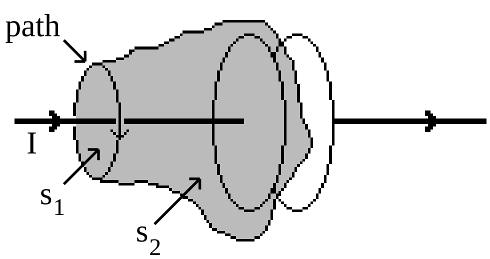
ferromagnetism: M > 0 and very large.

We have finally explained the common refrigerator magnet!

5. Induced Magnetic Fields

Are there other ways, besides currents, to create circulating magnetic fields? Yes! with changing E-fields.

Beyond the Mechanical Universe (vol. 39 Ch 30,31,32)



Here is an illustration of an ambiguity in Ampere's Law. Current is shown flowing through a capacitor as it charges. The path integral of B around the indicated path gives a definite value, but it is not clear what is meant by $i_{enclosed}$. Is it the current that crosses the plane surface s_1 or can it be the current the that crosses any surface that is bounded by the path? If so, the surface s_2 has no current crossing it. It can't matter which surface we pick, so Ampere's Law must contain another term on the right hand side to account for surfaces such as s_2. It is called the "displacement current" i_d. Now we can write,

$\oint \vec{B} \cdot d\vec{s} = \mu_o (i_{enclosed} + i_d)$ where $i_d = 0$ on s_1 and $i_d = I$ on s_2.

The origin of the displacement term can be understood by considering the fact that its units must also be units of current so, $i_d = \frac{dq}{dt}$. Since there is no charge flowing across s_2 perhaps this q is the charge on the capacitor q=CV. Assuming parallel plates, $q = \left(\varepsilon_o \frac{A}{d}\right)(Ed) = \varepsilon_o EA$ and the displacement current is

$i_d = \frac{d}{dt}(\varepsilon_o EA) = \varepsilon_o \frac{d}{dt}(EA) = \varepsilon_o \frac{d\Phi_e}{dt}$.

The corrected Ampere's Law is,

$$\oint \vec{B} \cdot d\vec{s} = \mu_o i_{enclosed} + \mu_o \varepsilon_o \frac{d\Phi_e}{dt} \qquad \text{Ampere's Law}$$

<u>Example 3:</u> A current of 5.00mA flows into a 10.0pF capacitor with circular plates of radius 2.00cm. Find (a)the displacement current, (b)the rate of change of the electric flux, (c)the rate of change of electric field, (d)the magnetic field 3.00cm from the center of the plates, and (e)the magnetic field 1.00cm from the center of the plates.

(a)The total displacement current must equal the actual current $i_d = 5.00\text{mA}$.

(b)The displacement current is proportional to the rate of change of flux,

$$i_d = \varepsilon_o \frac{d\Phi_e}{dt} \qquad \frac{d\Phi_e}{dt} = \frac{i_d}{\varepsilon_o} = \underline{\underline{5.65\text{x}10^8 \frac{\text{V m}}{\text{s}}}}.$$

(c)Using the definition of flux, $\frac{d\Phi_e}{dt} = \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$. Since the field between the plates is not a function of position, $\frac{d\Phi_e}{dt} = \frac{d}{dt}EA = \pi r^2 \frac{dE}{dt} \qquad \frac{dE}{dt} = \frac{1}{\pi r^2}\frac{d\Phi_e}{dt} = \underline{\underline{4.50\text{x}10^{11} \frac{\text{V}}{\text{m s}}}}$ (r = 2.00cm)

(d)Apply Ampere's Law to a 3.00cm radius circular path in a plane centered between the capacitor plates. There will be no real current enclosed only displacement current so

$$\oint \vec{B}\cdot d\vec{s} = \mu_o\varepsilon_o \frac{d\Phi_e}{dt} = \mu_o i_d.$$

By symmetry the field will be constant along this path

$$B\ 2\pi r = \mu_o i_d \qquad B = \frac{\mu_o i_d}{2\pi r} = \underline{\underline{3.33\text{x}10^{-8}\text{T}}}.$$

(e)When r = 1.00cm only part of the flux passes through the path. Using Ampere's Law,

$$\oint \vec{B}\cdot d\vec{s} = \mu_o i_{enclosed} + \mu_o\varepsilon_o \frac{d\Phi_e}{dt} \qquad B\ 2\pi r = \mu_o\varepsilon_o \frac{dE}{dt}\pi r^2 \qquad B = \frac{\mu_o\varepsilon_o r}{2}\frac{dE}{dt} = \underline{\underline{2.50\text{x}10^{-8}\text{T}}}.$$

In summary, the laws of electricity and magnetism have a nice symmetry. Just as changing magnetic fields produce circulating electric fields, changing electric fields make circulating magnetic fields.

<u>6. Maxwell's Equations</u>

We now know everything there is to know about the properties of the electric and magnetic fields. This knowledge is summarized by Maxwell's Equations,

Gauss's Law for Electricity	$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_o}$	Charge creates diverging electric fields.
Faraday's Law of Induction	$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Changing B's create circulating E's.
Gauss's Law for Magnetism	$\oint \vec{B}\cdot d\vec{A} = 0$	There are no magnetic monopoles.
Ampere's Law	$\oint \vec{B}\cdot d\vec{s} = \mu_o \frac{dq}{dt} + \mu_o\varepsilon_o \frac{d\Phi_e}{dt}$	Currents or changing E's make circulating B's.

<u>*Example 4:*</u> *Suppose a magnetic monopole is found experimentally. Fix Maxwell's Equations and find the SI units of magnetic charge.*

Gauss's Law for Magnetism must be fixed: $\oint \vec{B} \cdot d\vec{A} = \mu_o q_m$.

Since flowing electric charge creates magnetic field, it is reasonable to assume that flowing magnetic charge will create electric fields. Therefor, Faraday's Law also needs to be amended: $\oint \vec{E} \cdot d\vec{s} = \mu_o \frac{dq_m}{dt} - \frac{d\Phi_B}{dt}$.

Maxwell's Equations now exhibit a beautiful symmetry,

Gauss's Law for Electricity	$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_o}$	electric charge creates electric fields
Faraday's Law of Induction	$\oint \vec{E} \cdot d\vec{s} = \mu_o \frac{dq_m}{dt} - \frac{d\Phi_B}{dt}$	flowing magnetic charge or changing B-fields create E-fields
Gauss's Law for Magnetism	$\oint \vec{B} \cdot d\vec{A} = \mu_o q_m$	magnetic charge creates magnetic fields
Ampere's Law	$\oint \vec{B} \cdot d\vec{s} = \mu_o \frac{dq}{dt} + \mu_o \varepsilon_o \frac{d\Phi_e}{dt}$	flowing electric charge or changing E-fields create B-fields

The units of q_m can be found from Gauss's Law for Magnetism:

$$[q_m] = \frac{[B][A]}{[\mu_o]} = \frac{\left[\frac{\mu_o i}{\ell}\right][A]}{[\mu_o]} = \frac{[i][A]}{[\ell]} = [i][\ell] = \underline{\underline{A\ m}}$$

<u>**Chapter 32 - Summary**</u>

The Magnetic Moment of a Charge $\vec{\mu} = \frac{q}{2m}\vec{L}$

The Definition of Magnetization $\vec{M} \equiv \frac{\vec{\mu}}{vol}$

Maxwell's Equations

Gauss's Law for Electricity	$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_o}$
Faraday's Law of Induction	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
Gauss's Law for Magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$
Ampere's Law	$\oint \vec{B} \cdot d\vec{s} = \mu_o \frac{dq}{dt} + \mu_o \varepsilon_o \frac{d\Phi_e}{dt}$