Problem 9.12 – Waypoints

1. Start with $[L^2, z]$ and express L^2 in terms of its components.

2. Use the known commutators from eq. 4.122 to express the result in terms of four products of x and y with L_x and L_y. The result is two anti-commutators.

3. Eliminate the appropriate half of each anti-commutator using the appropriate commutator from eq. 4.122. A little algebra gives the stated result,

$$[L^2, z] = 2i\hbar\left(xL_y - yL_x - i\hbar z\right).$$

4. Write the two cyclic permutations of the stated result for $[L^2, x]$ and $[L^2, y]$. Eliminate the term that contains $i\hbar$ in these two results by using the commutator that the value equal to the $i\hbar$ term.

5. Finally, it is time to substitute the three commutators $[L^2, x]$, $[L^2, y]$, and $[L^2, z]$ into $\left[L^2, [L^2, z]\right]$. Simplify the resulting mess by explaining that $\vec{r} \cdot \vec{L}$ must equal zero which requires $xL_x+yL_y=-zL_z$.

6. Stay strong and you'll get the final result!