## Gauge Invariance Issues and the Aharanov-Bohm Effect

Replacing $\vec{p} \rightarrow \vec{p} + q\vec{A}$ in Schrodinger's Equation has some interesting consequences on the phase of the wave function. This comes about because the vector potential for a given magnetic field is not unique. Changing from one vector potential to another is called a "Gauge Transformation." Gauge transformations have a distinct effect on the wave function. The Aharonov-Bohm Effect illustrates some of these features. These results indicate a need to rethink our notion that charged particles are only affected by local electric and magnetic fields.

(a) Write the magnetic field (**B**) in terms of the vector potential (**A**). Show that the replacement,

$$\vec{A} \rightarrow \vec{A} + \nabla f$$

where f is any function of the coordinates and time, leave the fields unchanged.

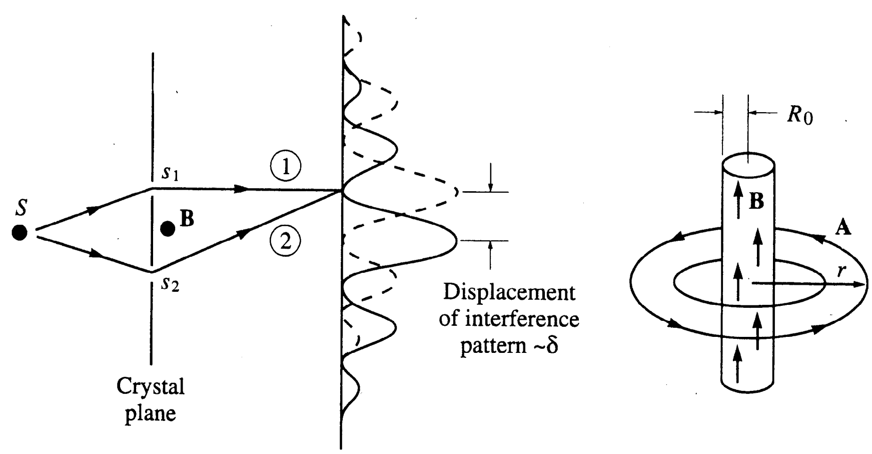
(b) Write time independent Schrodinger's Equation replacing $\vec{p} \rightarrow \vec{p} + q\vec{A}$. Call the solution, $\psi$.

(c) Write Schrodinger's Equation again replacing $\vec{A} \rightarrow \vec{A} + \nabla f$. Call the solution, $\psi'$. Show that $\psi' = \psi e^{i\Lambda(r)}$ satisfies Schrodinger's Equation for one particular value of $\Lambda$. Find this $\Lambda$.

(d) Explain the meaning of this result.

(e) Consider the physical situation shown below.

**FIGURE 14.2**
*The Aharonov-Bohm effect. (a) The path of the electron through a crystal (schematically represented by a double-slit) does not intersect the confined axial magnetic-field lines perpendicular to the plane of the paper shown separately in (b) where the vector potential of the field is also shown.*




(a) (b)

Show that the vector potential inside the solenoid can be expressed as $\vec{A} = \frac{B}{2}(\hat{k} \times \vec{r})$ and the vector potential outside is given by $\vec{A} = \frac{B}{2}(\hat{k} \times \vec{r})\frac{R_o^2}{r^2}$ where B is the magnetic field inside the solenoid by using it to find the fields inside and out. Note that $\vec{r} = x\hat{i} + y\hat{j}$ and has no z-component.

(f) Consider an electron outside a long solenoid where the magnetic field is zero. Show that the vector potential in such a region can be written as the gradient of a scalar function, g. Solve for g in terms of a line integral of this vector potential over path $s_1$. Use the result of part (c) to find the phase shift in the wave function for an electron taking path 1. Write this also for path 2.

(g) Find the phase difference between electrons traveling along the two paths. Note that the two paths form a closed loop. Use Stokes theorem to write the line integral in terms of a surface integral. Then complete the integration. Explain the meaning of your result.