Problem 6.6*

Given that a 2x2 portion of the unperturbed Hamiltonian matrix is degenerate, it might look like,

$$H^0 = \begin{pmatrix} E^0 & 0 \\ 0 & E^0 \end{pmatrix}.$$

(a)Write a set of orthonormal eigenvectors and their corresponding eigenvalues for H^0. Comment on the uniqueness of your choice.

The perturbation in general would add terms such as,

$$H' = \begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix}.$$

We wish to diagonalize the total Hamiltonian, so we just need to diagonalize H'.
(b)Find the eigenvalues of H' and show that they agree with eq. 6.27.

(c)Write the two possible eigenvectors of H' as $\psi_{\pm} \rightarrow \begin{pmatrix} \alpha_{\pm} \\ \beta_{\pm} \end{pmatrix}$ and find the conditions on α and β. Show that you get eq. 6.22 and eq. 6.24.

(d)Now looking at Problem 6.6 in the book, knowing the properties of Hermitian operators and the fact that $\psi_{\pm}$ is an eigenfunction of H', explain the answers to parts a, b, and c.