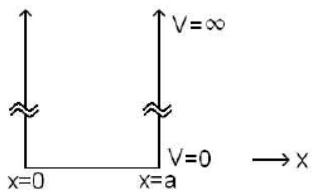
Quantum Mechanics Survey

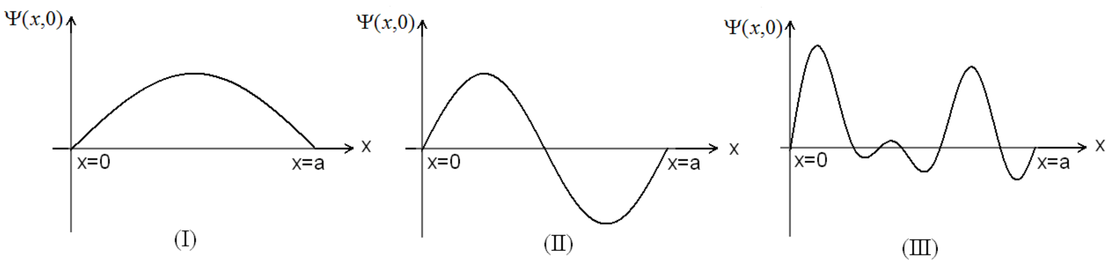
<u>Instructions for all of the questions</u>

* $\hat{x}$, $\hat{p}$ and $\hat{H}$ correspond to the position, momentum and Hamiltonian operators, respectively, for a given quantum system.
* A physical observable is "well-defined" when it has a definite value for a given wavefunction.
* "1D" is an abbreviation for "one dimensional". All questions refer to systems in one spatial dimension.
* The wavefunction at time *t* is written as $\Psi(x,t)$

System I: A particle interacts with a one-dimensional infinite square well of width a ($V(x)=0$ for $0 \le x \le a$ and $V(x)=+\infty$ otherwise) as shown. The stationary states are $\psi_n(x)=\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the allowed energies are $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where $n=1,2,3,\ldots,\infty$.	 Infinite Square Well

Questions 1 through 15 relate to System I, shown above:

1. Choose all of the following wave functions that are possible for System I at time *t*=0. $\Psi(x,0)$ and $d\Psi(x,0)/dx$ are both continuous and single-valued in the region $0<x<a$ for the states shown below.



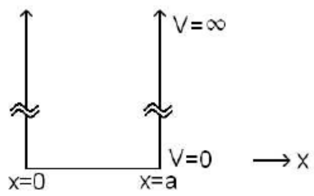
A. None of the above
B. All of the above
C. (I) only
D. (I) and (II) only
E. (I) and (III) only

2. Suppose that at time *t*=0, System I is in the first excited state. Choose all of the following expectation value(s) that depend on time.

(1) $\langle x\rangle$ (2) $\langle p\rangle$ (3) $\langle H\rangle$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. None of the above

System I: A particle interacts with a one-dimensional infinite square well of width a ($V(x)=0$ for $0 \le x \le a$ and $V(x)=+\infty$ otherwise) as shown. The stationary states are $\psi_n(x)=\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the allowed energies are $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where $n=1,2,3,...,\infty$.



Infinite Square Well

3. Consider the following wavefunction: $\Psi(x,0)=\sqrt{2/a}\sin(5\pi x/a)$. Which one of the following is the probability density $|\Psi(x,t)|^2$, at time t ?

A. $|\Psi(x,t)|^2=\frac{2}{a}\sin^2(5\pi x/a)\cos^2(E_5 t/\hbar)$

B. $|\Psi(x,t)|^2=\frac{2}{a}\sin^2(5\pi x/a)\exp(-i2E_5 t/\hbar)$

C. $|\Psi(x,t)|^2=\frac{2}{a}\sin^2(5\pi x/a)\sin^2(E_5 t/\hbar)$

D. $|\Psi(x,t)|^2=\frac{2}{a}\sin^2(5\pi x/a)$ which is time-independent.

E. None of the above.

4. Consider the following wavefunction: $\Psi(x,0)=A\sin^5(\pi x/a)$, where A is a suitable normalization constant. Which one of the following is the probability density $|\Psi(x,t)|^2$, at time t ?

A. $|\Psi(x,t)|^2=|A|^2\sin^{10}(\pi x/a)\cos^2(E_5 t/\hbar)$

B. $|\Psi(x,t)|^2=|A|^2\sin^{10}(\pi x/a)\exp(-i2E_5 t/\hbar)$

C. $|\Psi(x,t)|^2=|A|^2\sin^{10}(\pi x/a)\sin^2(E_5 t/\hbar)$

D. $|\Psi(x,t)|^2=|A|^2\sin^{10}(\pi x/a)$ which is time-independent.

E. None of the above.

5. Suppose the particle for System I is in the ground state with wavefunction $\psi_1(x)$. Which one of the following is the probability that the particle will be found in a narrow range between x and $x+dx$?

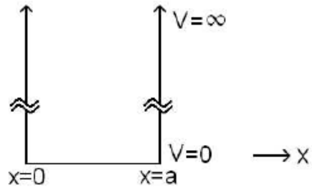
A. $|\psi_1(x)|^2 dx$

B. $x|\psi_1(x)|^2 dx$

C. $\int_x^{x+dx} x|\psi_1(x)|^2 dx$

D. $\int_{-\infty}^{+\infty} x|\psi_1(x)|^2 dx$

E. None of the above.

System I: A particle interacts with a one-dimensional infinite square well of width a ($V(x)=0$ for $0 \le x \le a$ and $V(x)=+\infty$ otherwise) as shown. The stationary states are $\psi_n(x)=\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the allowed energies are $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where $n=1,2,3,...,\infty$.	 Infinite Square Well

6. Choose all of the following statements that are correct for System I. In all the options, A_n are suitable expansion coefficients such that the wavefunction is normalized and at least <u>two</u> A_n are nonzero.

   (1) $\Psi(x,0)=\sum_n A_n\psi_n(\mathrm{x})$ is a possible wavefunction for the particle at time t=0.

   (2) $\Psi(x,0)=\sum_n A_n\psi_n(\mathrm{x})$ satisfies the time independent Schrödinger equation, $\hat{H}\Psi(x,0)=E\Psi(x,0)$.

   (3) $\Psi(x,t)=\sum_n A_n\psi_n(\mathrm{x})\exp(-iE_n t/\hbar)$ satisfies the time dependent Schrödinger equation, $i\hbar\frac{\partial\Psi(x,t)}{\partial t}=\hat{H}\Psi(x,t)$

A. 1 only B. 2 and 3 only C. 1 and 2 only D. 1 and 3 only E. All of the above

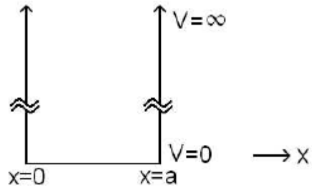
7. The wavefunction for System I at time t=0 is $\sqrt{2/7}\psi_1(x)+\sqrt{5/7}\psi_2(x)$ when you perform a measurement of energy. The energy measurement yields $4\pi^2\hbar^2/2ma^2$. Which one of the following is the spatial part of the <u>normalized</u> wavefunction (excluding the time part) after the energy measurement? (Ignore the overall phase of the wavefunction.)

   (1) $\psi_2(x)$

   (2) $\sqrt{5/7}\psi_2(x)$

   (3) $\sqrt{2/7}\psi_1(x)+\sqrt{5/7}\psi_2(x)$

A. 1 only
B. 2 only
C. 3 only
D. Depends on how long you wait after the energy measurement. At the instant energy is measured, it will be (1) but a long time after that it will be (3).
E. Depends on how long you wait after the energy measurement. At the instant energy is measured, it will be (2) but a long time after that it will be (3).

System I: A particle interacts with a one-dimensional infinite square well of width a ($V(x)=0$ for $0 \le x \le a$ and $V(x)=+\infty$ otherwise) as shown. The stationary states are $\psi_n(x)=\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the allowed energies are $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where $n=1,2,3,...,\infty$.	 Infinite Square Well

8. Consider a wavefunction for System I given by $\Psi(x,t=0)=\sqrt{4/7}\psi_1(x)+\sqrt{3/7}\psi_2(x)$.

   Which one of the following statements is true about the measurement of energy for this system?

A. The only possible value of energy measured will be $4E_1/7+3E_2/7$.

B. The only possible values measured will be the ground state energy E_1 or the first excited state energy E_2.

C. The possible values of energy measured will be $E_n=n^2\pi^2\hbar^2/(2ma^2)$ (where n is any positive integer) but the probability of measuring E_1 is largest.

D. The possible values of energy measured will be $E_n=n^2\pi^2\hbar^2/(2ma^2)$ (where n is any positive integer) and all E_n are equally probable.

E. None of the above.

9. Consider the following wavefunction for System I: $\Psi(x,t=0)=\frac{1}{\sqrt{3}}\psi_1(\mathrm{x})+i\sqrt{\frac{2}{3}}\psi_2(\mathrm{x})$.

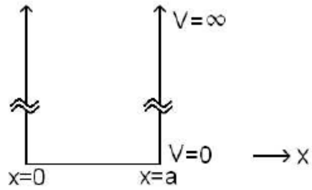
   Choose all of the following statements that are correct about the **expectation value of the energy** of the system at time t=0.

   (1) $\langle E\rangle=\frac{1}{3}E_1-\frac{2}{3}E_2$

   (2) $\langle E\rangle=\frac{1}{3}E_1+\frac{2}{3}E_2$

   (3) $\langle E\rangle=\int_0^a\Psi^*(x,0)\hat{H}\Psi(x,0)dx$

A. 1 only
B. 2 only
C. 3 only
D. 1 and 3 only
E. 2 and 3 only

System I: A particle interacts with a one-dimensional infinite square well of width a ($V(x)=0$ for $0 \le x \le a$ and $V(x)=+\infty$ otherwise) as shown. The stationary states are $\psi_n(x)=\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the allowed energies are $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where $n=1,2,3,\ldots,\infty$.	 Infinite Square Well

10. Consider the following wavefunction for System I: $\Psi(x,t=0)=\frac{1}{\sqrt{3}}\psi_1(\mathrm{x})+i\sqrt{\frac{2}{3}}\psi_2(\mathrm{x})$.

Choose all of the following statements that are correct about the **expectation value of the energy** of the system at time t>0.

(1) $\langle E\rangle=\frac{1}{3}E_1e^{-iE_1t/\hbar}-\frac{2}{3}E_2e^{-iE_2t/\hbar}$

(2) $\langle E\rangle=\frac{1}{3}E_1e^{-iE_1t/\hbar}+\frac{2}{3}E_2e^{-iE_2t/\hbar}$

(3) $\langle E\rangle=\int_0^a\Psi^*(x,t)\hat{H}\Psi(x,t)dx$

A. None of the above
B. 1 only
C. 2 only
D. 3 only
E. 1 and 3 only

11. Consider the following wavefunction for System I at time $t=0$: $\Psi(x,0)=Ax(a-x)$ for $0\le x\le a$ and $\Psi(x,0)=0$ otherwise. A is a normalization constant. Which one of the following expressions correctly represents the probability of measuring the energy E_n in the state $\Psi(x,0)$?

A. $\left|\int_0^a\psi_n^*(x)\hat{H}\Psi(x,0)dx\right|^2$

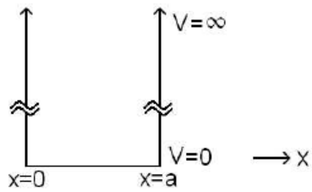
B. $\left|\int_0^a\psi_n^*(x)\Psi(x,0)dx\right|^2$

C. $\left|\psi_n^*(x)\hat{H}\Psi(x,0)\right|^2$

D. $\left|\psi_n^*(x)\Psi(x,0)\right|^2$

E. $\left|\Psi(x,0)\right|^2$

System I: A particle interacts with a one-dimensional infinite square well of width a ($V(x)=0$ for $0 \le x \le a$ and $V(x)=+\infty$ otherwise) as shown. The stationary states are $\psi_n(x)=\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the allowed energies are $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where $n=1,2,3,\ldots,\infty$.



Infinite Square Well

12. Particle A is in a 1D infinite square well (system I) and particle B is in a separate, identical well. At time *t*=0, particle A is in the state $\frac{1}{\sqrt{2}}(\psi_1+\psi_2)$, and particle B is in the state $\frac{1}{\sqrt{2}}(\psi_1+i\psi_2)$. Choose all of the following statements that are correct at a given time *t*>0.
    (1) The particles A and B have the same expectation value of position at time *t*.
    (2) The particles A and B have the same expectation value of momentum at time *t*.
    (3) The particles A and B have the same expectation value of energy at time *t*.

A. 1 only B. 2 only C. 3 only D. All of the above E. None of the above

13. At time $t=0$, the state for System I is $\frac{1}{\sqrt{2}}(\psi_1+\psi_2)$. We first measure the position of the particle at time $t=0$ and obtain the result x_0. **Immediately after the position measurement**, we measure the energy. What possible result(s) can we obtain for the energy measurement?

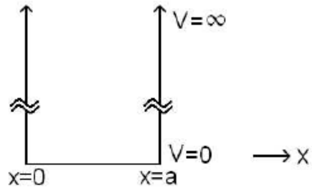
A. We can only measure either E_1 or E_2.

B. We can obtain one of the energy values $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where *n* can be an arbitrarily large integer.

C. We can only measure $\frac{1}{2}(E_1+E_2)$.

D. We may measure any energy $E=\sum_{n=1}^{\infty}c_nE_n$ where c_n are coefficients so that $\sum_{n=1}^{\infty}|c_n|^2=1$.

E. None of the above.

System I: A particle interacts with a one-dimensional infinite square well of width a ($V(x)=0$ for $0 \le x \le a$ and $V(x)=+\infty$ otherwise) as shown. The stationary states are $\psi_n(x)=\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the allowed energies are $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$ where $n=1,2,3,...,\infty$.	 Infinite Square Well

14. At time *t*=0, consider the function $\Psi(x,t=0)=A\sin^2(\pi x/a)$ for $0 \le x \le a$ and $\Psi(x,0)=0$ otherwise. A is a suitable normalization constant. Choose all of the following statements that are correct about whether this function is a possible wavefunction for System I at time *t*=0.

(1) It is not a possible wavefunction because it is not in the form $A\sin(n\pi x/a)$ where n=1, 2, 3, … ∞.

(2) It is not a possible wavefunction because it does not satisfy the time independent Schrödinger equation $H\Psi(x,0)=E\Psi(x,0)$.

(3) It is a possible wavefunction for two particles instead of a single particle.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. none of the above

15. A particle is initially in the ground state of an infinite square well with boundaries between $0 \le x \le a$. At time *t*=0 the width of the well suddenly increases to 2*a* (boundaries between $0 \le x \le 2a$) so that the wavefunction does not have time to change during the increase in width. Which one of the following statements is correct at time *t* >0?

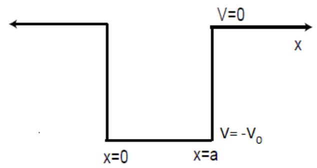
A. The wavefunction of the particle will stay in the ground state of the initial well for all future times even though the well has expanded.

B. The wavefunction of the particle will evolve to the ground state wavefunction of the new well and stay in that state.

C. The wavefunction of the particle will change to the first excited state of the new well and stay in that state.

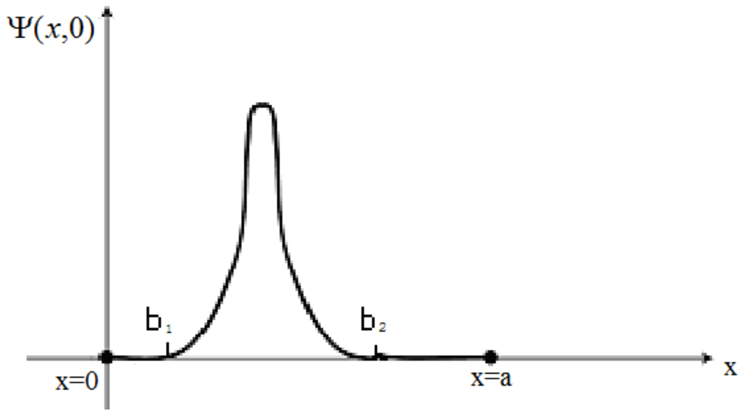
D. The probability density function of the particle will change with time for all *t* >0.

E. None of the above.

System II: A particle interacts with a one-dimensional finite square well of width a ($V(x) = -V_0$ for $0 \le x \le a$ and $V(x) = 0$ otherwise) as shown.	 Finite Square Well

Questions 16 through 19 relate to System II as shown above:

16. Choose all of the following statements that are correct about the wavefunction shown below for system II at time t=0. $\Psi(x,0)$ and $d\Psi(x,0)/dx$ are continuous and single-valued everywhere. The wavefunction $\Psi(x,0)$ is zero in the regions $x < b_1$ and $x > b_2$. Assume that the area under the $|\Psi(x,0)|^2$ curve is 1.

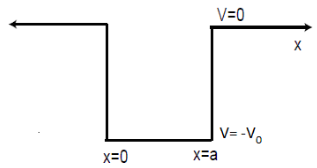


(1) It is a possible wavefunction.
(2) It is not a possible wavefunction because it does not satisfy the boundary conditions. Specifically, it goes to zero inside the well.
(3) It is not a possible wavefunction because the probability of finding the particle outside the finite square well is zero but quantum mechanically it must be nonzero.

A. 1 only B. 2 only C. 3 only D. 2 and 3 only E. None of the above

17. Suppose you perform a measurement of the position of the particle when it is in the first excited state of system II. Choose all of the following statements that are correct about this experiment:
(1) Right after the position measurement, the wavefunction will be peaked about a particular value of position.
(2) A long time after the position measurement, the wavefunction will go back to the first excited state wavefunction.
(3) The wavefunction will not go back to the first excited state wavefunction even if you wait for a long time after the position measurement.

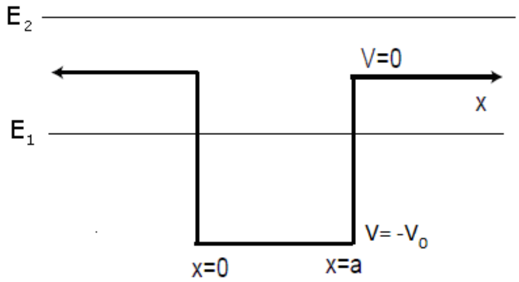
A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. 1 and 3 only

System II: A particle interacts with a one-dimensional finite square well of width a $(V(x) = -V_0$ for $0 \le x \le a$ and $V(x) = 0$ otherwise) as shown.	 Finite Square Well

18. Choose all of the following statements that are correct for System II, regardless of whether it is in a <u>bound state or a scattering state</u>.
    (1) The particle may be found outside the region between *x*=0 and *x*=*a*.
    (2) The energy levels of the system are discrete.
    (3) The stationary state wavefunctions may or may not be normalizable but we can make a normalizable wavefunction by using their linear superposition.

A. 1 only B. 2 only C. 3 only D. 1 and 3 only E. All of the above

19. A particle interacts with a finite square well of width a as shown below. Two possible energies E_1 and E_2 are such that $-V_0 < E_1 < 0$ and $E_2 > 0$. Choose all of the following statements that are correct about a particle in a <u>bound or a scattering</u> state.



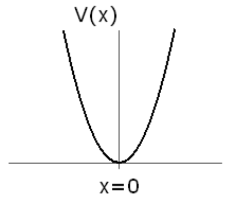
(1) A single particle in a given stationary state can have an energy E_1 inside the well and have a different energy E_2 outside the well.

(2) Whether the particle is in a bound or a scattering state depends on the location of the particle if the energy of the particle is E_2.

(3) Whether the particle is in a bound or a scattering state depends on the location of the particle if the energy of the particle is E_1.

A. 1 only B. 2 only C. 3 only D. 2 and 3 only E. none of the above

System III: A particle with mass m interacts with a one-dimensional simple harmonic oscillator well $V(x) = Kx^2/2$ as shown. The stationary states are $\psi_n(x)$ and the allowed energies are $E_n = (n+1/2)\hbar\omega$ where $n = 0,1,2,3,...,\infty$ and $\omega = \sqrt{K/m}$.



Simple Harmonic Oscillator

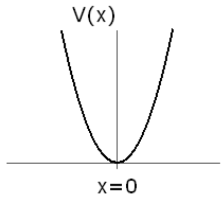
Questions 20-23 refer to System III shown above:

20. Choose all of the following statements that are correct about system III with a definite energy:
    (1) Although classically the particle has the highest probability of being found at its (classical) turning points, for the quantum mechanical <u>ground state</u>, it is most likely to be found at the center of the well.
    (2) The probability of finding the particle at the center of the well is zero for the first excited state.
    (3) The probability of finding the particle is zero in the regions where the energy E of the particle is less than $V(x)$.

A. 1 only B. 2 only C. 1 and 2 only D. 2 and 3 only E. all of the above

21. Consider two particles A and B in independent 1D simple harmonic oscillator potential energy wells, both identical to System III. Choose all of the following statements that are correct.
    (1) If the expectation value of the momentum of particle A is larger than that for particle B ($\langle p_A \rangle > \langle p_B \rangle$), the uncertainty in the position of particle A is smaller than that for particle B ($\Delta x_A < \Delta x_B$).
    (2) If the expectation value of the momentum of particle A is larger than that for particle B ($\langle p_A \rangle > \langle p_B \rangle$), the uncertainty in the momentum of particle A is larger than that for particle B ($\Delta p_A > \Delta p_B$).
    (3) If the expectation value of the position of particle A is larger than that for particle B ($\langle x_A \rangle > \langle x_B \rangle$), the expectation value of the momentum of particle A is smaller than that for particle B ($\langle p_A \rangle < \langle p_B \rangle$).

A. None of the above B. 1 only C. 2 only D. 3 only E. 1 and 3 only

System III: A particle with mass m interacts with a one-dimensional simple harmonic oscillator well $V(x) = Kx^2/2$ as shown. The stationary states are $\psi_n(x)$ and the allowed energies are $E_n = (n+1/2)\hbar\omega$ where $n = 0,1,2,3,...,\infty$ and $\omega = \sqrt{K/m}$.	 Simple Harmonic Oscillator

22. You prepare System III in an unknown superposition of ψ_0 and ψ_3. Choose all of the following statements that are correct about how to experimentally estimate the probability distribution of the possible energies that can be measured in this state:
    (1) Perform repeated measurements of the energy of the same particle a very large number of times to obtain the probability distribution.
    (2) Perform a measurement of the particle's energy. Then, wait for a long time before measuring the energy again. Repeat this process a very large number of times on that particle to obtain the probability distribution.
    (3) Perform a measurement of the particle's energy. Then, prepare the system again such that it has the original unknown superposition wave function. Then, measure the energy again. Repeat this process of preparation and measurement a very large number of times to obtain the probability distribution.

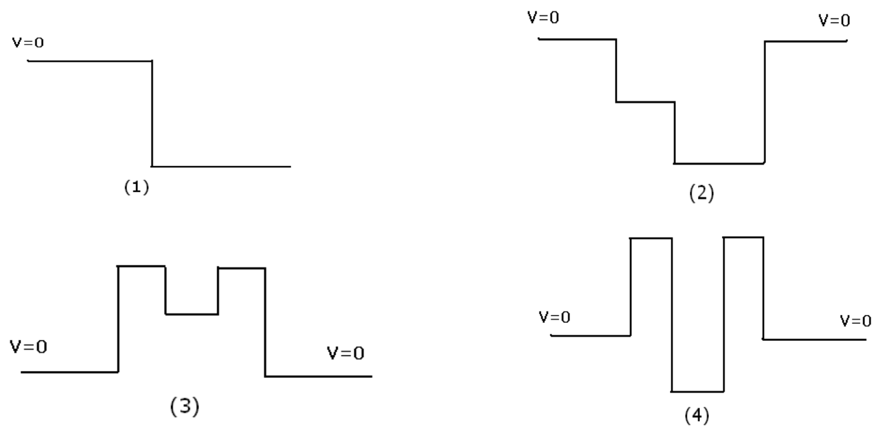
A. 1 only B. 2 only C. 3 only D. 2 and 3 only E. All of the above

23. Suppose at time t=0, System III is in the state $\frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$. Choose all of the following expectation values that **depend** on time.

    (1) $\langle x \rangle$ (2) $\langle p \rangle$ (3) $\langle H \rangle$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. all of the above

24. Choose all of the 1D potential energies shown in the figures below that allow both bound and scattering states.



A. 1 only B. 2 only C. 2 and 4 only D. 3 and 4 only E. 2, 3 and 4 only

25. e^{ikx} is a stationary state wavefunction for a <u>free</u> particle in one dimension with momentum $p = \hbar k$. If the wavefunction of a <u>free</u> particle is $e^{ikx} + e^{-ikx}$, choose all of the following statements that are correct. (Ignore the wavefunction normalization issues in this problem.)

(1) The particle is still in a stationary state.
(2) The expectation value of the momentum of the particle is zero.
(3) The expectation value of the energy of the particle is zero.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. 1 and 3 only

26. Choose all of the following equations that are correct for a quantum particle interacting with a 1D potential energy $V(x)$. (If you have learned both the Schrödinger & Heisenberg formalisms of quantum mechanics, use the Schrödinger formalism.)

(1) $\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}$

(2) $\frac{d\hat{p}}{dt} = -\frac{\partial V(\hat{x})}{\partial x}$

(3) $\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$

A. 3 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. All of the above

27. Choose all of the following one dimensional Hamiltonian operators that have <u>ONLY</u> a discrete energy spectrum.

(1) $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ for all x.

(2) $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{K}{2}x^2$ for all x where K is a positive constant.

(3) $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$, $V(x) < 0$ (but finite) when $0 < x < a$ and $V(x) = 0$ otherwise.

A. 1 only B. 2 only C. 2 and 3 only D. all of the above E. none of the above

28. Choose all of the following statements that are correct:

(1) The stationary states refer to the eigenstates of any operator corresponding to any physical observable.
(2) In an isolated system, if a particle is in a position eigenstate (has a definite value of position) at time t=0, the position of the particle is well-defined at all times t >0.
(3) In an isolated system, if a system is in an energy eigenstate (it has a definite energy) at time t=0, the energy of the particle is well-defined at all times t >0.

A. 1 only B. 3 only C. 1 and 3 only D. 2 and 3 only E. All of the above

29. Choose all of the following statements that are correct about the Hamiltonian operator for a quantum system with potential energy $V=V(x)$.
    (1) The Hamiltonian governs the time evolution of the quantum system.
    (2) The Hamiltonian determines the form of the stationary state wavefunctions $\psi_n(x)$.
    (3) The Hamiltonian determines the shape of the position eigenfunction (wavefunction for which the position of the particle has a definite value).

A. 1 only B. 2 only C. 1 and 2 only D. 2 and 3 only E. All of the above

30. A particle is interacting with a 1D potential energy well $V(x)$. If $V(x)$ is an even function, choose the correct statement about any **possible wavefunction** $\Psi(x,t)$ (not necessarily a stationary state) for the system at a specific time t.

A. $\Psi(x,t)$ must be even.

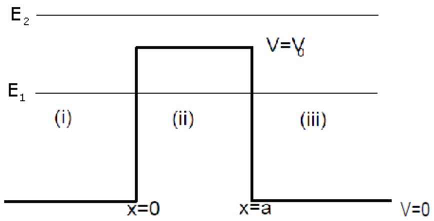
B. $\Psi(x,t)$ must be odd.

C. $\Psi(x,t)$ must be symmetric, but the symmetry axis is not necessarily about $x=0$.

D. $\Psi(x,t)$ must be even or odd, and no other possibility is allowed.

E. None of the above

31. A particle interacts with a 1D finite potential energy barrier as shown below. Two possible energies E_1 and E_2 are such that $0 < E_1 < V_0$ and $E_2 > V_0$. Choose all of the following statements that are correct.



(1) A particle with energy E_1 is in a bound state between $x=0$ and $x=a$ and in a scattering state elsewhere.

(2) A particle with energy E_2 can be in a bound or a scattering state depending on its location.

(3) The energy E_2 corresponds to a particle in a scattering state and the energy E_1 corresponds to a particle in a bound state.

A. none of the above B. 1 only C. 2 only D. 3 only E. 1 and 2 only