An Old Pilot's Yarn

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As physics teachers, we often state platitudes about physics being all around us and we hope that our students take away from our classes the idea that physics can help them make sense of the world regardless of what their future holds. One of the greatest joys of a long career as a teacher is the "blast from the past" that occurs when a former student drops you a line. It is even more of a treat when he wants to discuss a feature of his life that he has a feeling can be explained by physics. It was my good fortune to have the following problem presented to me by a former student. Since it illustrates some basic vector properties, you might find it of interest for your students.

Pilots have an old yarn that states the air speed of a plane can be found by measuring the ground speed of the plane when the air speed is directed along each cardinal (north, south, east, and west) and averaging the four values. This is a way to check the accuracy of the air speed indicator on the plane. The ground speed, V, is the magnitude of the velocity of the plane, $\vec{V}$, with respect to the ground. These days this measurement is easily accomplished by use of GPS. The air speed, v, is the magnitude of the velocity of the plane, $\vec{v}$, with respect to the air, and it is related to the ground speed by

$$\vec{V} = \vec{v} + \vec{w},$$

where $\vec{w}$ is the velocity of the wind with respect to the ground. The four vector diagrams for each segment of the flight are shown in Fig. 1. The wind direction is indicated by the angle α measured from the north.

Using the law of cosines, we obtain four equations:[1]

$$V_s^2 = v^2 + w^2 - 2vw \cos\alpha, \tag{1}$$

$$V_w^2 = v^2 + w^2 - 2vw \sin\alpha, \tag{2}$$

$$V_n^2 = v^2 + w^2 + 2vw \cos\alpha, \tag{3}$$

$$V_e^2 = v^2 + w^2 + 2vw \sin\alpha. \tag{4}$$

Notice that if the goal were just to find the three unknowns v, w, and α, given the measured values V_n, V_e, V_s, and V_w, there is more information here than required. Since there are only three unknowns, only three of these equations are needed.[2] However, the algebra needed to solve for the three unknowns is somewhat unpleasant, and adding a fourth measurement to average may reduce the uncertainty in the result.

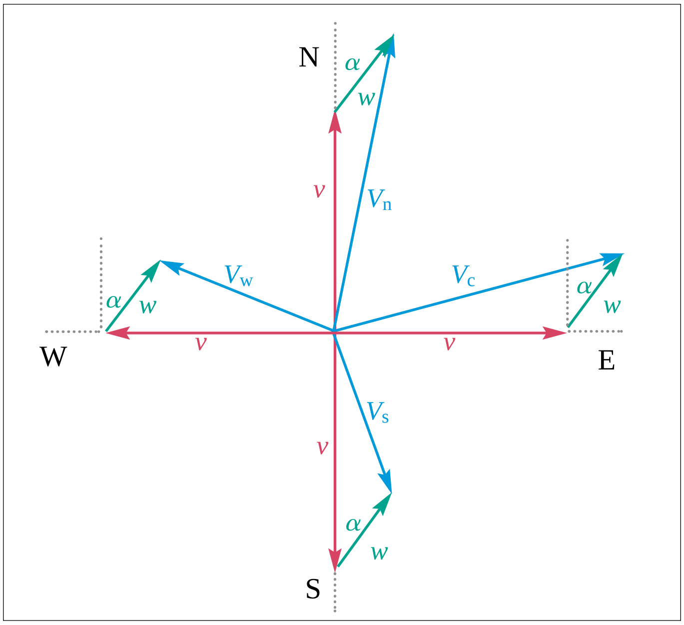

Fig. 1. Vector diagram illustrating the four cases, one for each cardinal.

Summing the four equations gives

$$V_n^2 + V_e^2 + V_s^2 + V_w^2 = 4v^2 + 4w^2 . \qquad (5)$$

Since the wind speed is generally small compared to the air speed for cases where conducting this experiment is safe,

$$4v^2 \approx V_n^2 + V_e^2 + V_s^2 + V_w^2 . \qquad (6)$$

Finally,

$$v \approx \sqrt{\tfrac{1}{4}(V_n^2 + V_e^2 + V_s^2 + V_w^2)}. \qquad (7)$$

We see that, as is the case for most old yarns, it is only a good approximation to the truth. The air speed isn't the average of the four readings; instead it is approximately the root-mean-square of the measurements. The average is a good approximation to the rms value because the four measurements differ only slightly due to the relatively small wind speed.

Acknowledgment

The author wishes to thank his former student Steve Conley for bringing this problem to his attention.

References

1. Note that cos $(\pi/2 - \alpha)$ = sin α, cos $(\pi - \alpha)$ = –cos α and cos $(\pi/2 + \alpha)$ = –sin α.
2. G.V. Lewis, "A Flight Test Technique Using GPS for Position Error Correction Testing" (National Test Pilot School, Mojave, CA); http://www.ntps.com/gps-pec.pdf.