The Anatomy of a Pitch: Doing Physics with PITCHf/x Data

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On Aug. 7, 2007, Barry Bonds of the San Francisco Giants was at bat waiting for a 3-2 pitch from Mike Bacsik of the Washington Nationals. The ball left the pitcher's hand at 84.7[1] mph and arrived at home plate traveling 77.2 mph. It was within 0.2 in of the center of home plate and 3.213 ft above the ground when Bonds swung and hit his 756th home run, making him the all-time leader for homers in a career.[2] Thanks to a company called Sportvision[3] and Major League Baseball,[4] you can get kinematic data on any pitch thrown. Just think of the interesting and realistic physics problems you can generate from such a rich data set!

Video technology to track pitches started to be installed in major league ballparks in 2005. The project was completed in 2007 and the system is called PITCHf/x.[5] PITCHf/x calculates the position versus time for each pitch using input from dedicated digital cameras. Sportvision claims to "measure the position of the baseball to better than ½ inch to the exact location of the ball."[3] The system uses this trajectory data to perform a least-squares fit to the initial position, initial velocity, and acceleration along each axis. Note that the acceleration is assumed constant over the entire flight of the pitch. This assumption leads to equations that claim to reproduce the actual trajectory. Keep in mind that the data are processed in real time during the game. Information on each pitch is available in a fraction of a second and is displayed in the ballpark and on MLB Gameday.[4]

The fabulous thing for you and your students is that the data are provided at no cost, by Major League Baseball[6] (although it is copyrighted). Alan Nathan has a good website to describe how to download the data and what the quantities actually mean.[7] Another way to get the data is from Dan Brooks' website PITCHf/x Tool.[8] The easiest way to get a set of data from one of several famous pitches is at the author's website (http:/ phys.csuchico.edu/baseball/kinematics.html).

While you or your students might find another pitch more interesting, the pitch described above will be used here to illustrate some of the physics you can do. Table I contains the information about the pitch Bonds hit.[9] Figure 1 illustrates the coordinate

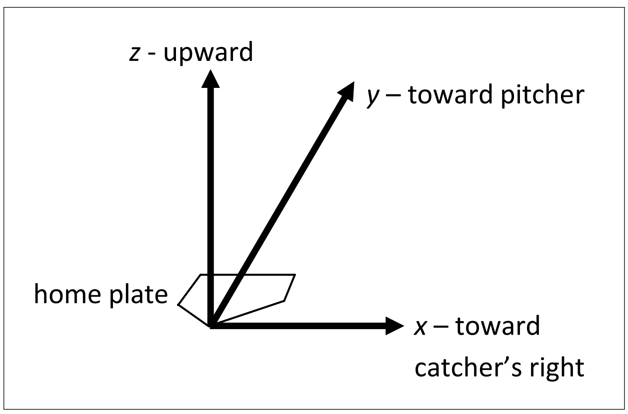

Fig. 1. The coordinate system has its origin at the back point of home plate on the ground. The *x*-axis points to the catcher's right. The *y*-axis is toward the pitcher. The *z*-axis is oriented upward.

Table I. The names of each quantity, its value, its units, and a brief description. For more complete information, see Alan Nathan's website.[7]

No.	Quantity	Value	Units	Descriptions
1	des	In play, run(s)		A comment on the action resulting from the pitch.
2	type	X		B=ball, S=strike, X=in play
3	id	371		Code indicating pitch number
4	*x* =	112.45	pixels	*x*-pixel at home plate
5	*y* =	131.24	pixels	*z*-pixel at home plate (yes, it is *z*)
6	start_speed	84.1	mph	Speed at *y*0=50ft
7	end_speed	77.2	mph	Speed at the front of home plate *y*=1.417ft
8	sz_top	3.836	ft	The *z*-value of the top of the strike zone as estimated by a technician
9	sz_bot	1.79	ft	The *z*-value of the bottom of the strike zone as estimated by a technician
10	pf*x*_*x*	8.68	in	A measure of the "break" of the pitch in the *x*-direction.
11	pf*x*_*z*	9.55	in	A measure of the "break" of the pitch in the *y*-direction.
12	p*x*	-0.012	ft	Measured *x*-value of position at the front of home plate (*y* = 1.417 ft)
13	p*z*	2.743	ft	Measured *z*-value of position at the front of home plate (*y* = 1.417 ft)
14	*x*0	1.664	ft	Least squares fit (LSF) value for the x-position at *y* = 50 ft
15	*y*0	50.0	ft	Arbitrary fixed initial *y*-value
16	*z*0	6.597	ft	LSF value for the *z*-position at *y* = 50 ft
17	v*x*0	-6.791	ft/s	LSF value for the *x*-velocity at *y* = 50 ft
18	v*y*0	-123.055	ft/s	LSF value for the *y*-velocity at *y* = 50 ft
19	v*z*0	-5.721	ft/s	LSF value for the *z*-velocity at *y* = 50 ft
20	a*x*	13.233	ft/s/s	LSF value for the *x*-acceleration assumed constant throughout the pitch.
21	a*y*	25.802	ft/s/s	LSF value for the *y*-acceleration assumed constant throughout the pitch.
22	a*z*	-17.540	ft/s/s	LSF value for the *z*-acceleration assumed constant throughout the pitch.
23	break_*y*	25.2	ft	Another measure of the "break." See Nathan's website for an explanation.
24	break_angle	-32.1	deg	Another measure of the "break." See Nathan's website for an explanation.
25	break_length	5.9	in	Another measure of the "break." See Nathan's website for an explanation.

system used. The origin is at the back point of home plate on the ground. The x-axis points to the catcher's right when he is facing the pitcher. The y-axis points directly toward the pitcher so the y-component of the pitched ball's velocity is always negative. The z-axis is oriented upward.

Let's start by finding the initial speed of the ball (at y = 50.0 ft). In three dimensions the initial speed is the magnitude of the initial velocity vector. Since the components are listed in the table (items 17, 18, and 19), we take the square root of the sum of their squares,

$$v_0 = \sqrt{v_{ox}^2 + v_{oy}^2 + v_{oz}^2} \tag{1}$$

$$v_0 = 123.375 \text{ ft/s} = 84.1 \text{ mph.}^{10} \tag{2}$$

This agrees precisely with the PITCHf/x value listed in Table I (item 6).

We can find the components of the final velocity of the pitch when it reaches the front of home plate (y = 1.417 ft) by using the kinematic equations. Since we know the initial and final y-values, we can get the y-component of the velocity using the kinematic equation,

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \Rightarrow v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -112.408 \text{ ft/s.} \tag{3}$$

Note that we want the negative value of the root to agree with the coordinate system. The time of flight must be found to get the other velocity components. Using another kinematic equation,

$$v_y = v_{0y} + a_y t \Rightarrow t = \tfrac{v_y - v_{0y}}{a_y} = 0.4127 \text{ s.} \tag{4}$$

Having the time of flight and using kinematic equations for the other two axes,

$$v_x = v_{0x} + a_x t = -1.330 \text{ ft/s.} \tag{5}$$

$$v_z = v_{oz} + a_z t = -12.960 \text{ ft/s}. \tag{6}$$

The final velocity vector is,

$$\mathbf{v} = (-1.330 \text{ ft/s})\hat{\imath} + (-112.408 \text{ ft/s})\hat{\jmath} + (-12.960 \text{ ft/s})\hat{k}\,. \tag{7}$$

Calculating the final speed,

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = 113.160 \text{ ft/s} = 77.2 \text{ mph,} \tag{8}$$

again in agreement with Table I (item 7).

A typical batter doesn't get a sense of the motion of the pitch until the ball is about 40 ft away from home plate. Let's find the time it takes the ball to get to y = 40 ft and the x- and z-components of the position and velocity when it gets there. Using the kinematic equation,

$$y = y_0 + v_{0y}t_{40} + \tfrac{1}{2}a_y t_{40}^2 \Rightarrow t_{40} = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 2a_y(y_0 - y)}}{a_y} = 0.08197 \text{ s,} \tag{9}$$

where we have used the minus sign. The x-position and velocity can now be found,

$$x_{40} = x_0 + v_{0x}t_{40} + \tfrac{1}{2}a_x t_{40}^2 = 1.152 \text{ ft} \tag{10}$$

$$v_{x40} = v_{0x} + a_x t_{40} = -5.706 \text{ ft/s,} \tag{11}$$

as can the z-position and velocity,

$$z_{40} = z_0 + v_{0z}t_{40} + \tfrac{1}{2}a_z t_{40}^2 = 6.069 \text{ ft.} \tag{12}$$

$$v_{z40} = v_{0z} + a_z t_{40} = -7.159 \text{ ft/s.} \tag{13}$$

Now that the batter has a sense of the position and velocity of the ball, he can begin to plan his swing. The batter has plenty of experience dealing with the force that the air exerts on the ball due to the drag of air resistance. Therefore, the batter might expect the ball to arrive at home plate at the usual rate determined by the actual y-component of the velocity he can estimate at y = 40 ft and typical air resistance. Therefore, time of flight from y = 40 ft can be found by subtracting the total time from the time to get to y = 40 ft,

$$t_{\mathrm{h}} = t - t_{40} = 0.4127 - 0.08197 = 0.3307 \text{ s.} \tag{14}$$

The batter has a much more difficult time estimating the spin the pitcher put on the ball. The spin causes the air to exert additional forces on the ball due to the Magnus effect. Assuming the air had no effect on the motion of the ball in the x- and z-directions starting at the point y = 40 ft, we can find the x- and z-positions of the ball when it gets to the front of home plate. Along the x-direction there would be no acceleration,

$$x_{\text{noair}} = x_{40} + v_{x40} t_{\text{h}} + \tfrac{1}{2} a_x t_{\text{h}}^2 \Rightarrow x_{\text{noair}} = -0.735 \text{ ft.} \quad (15)$$

Along the z-axis there would only be gravity,

$$z_{\text{noair}} = z_{40} + v_{z40} t_{\text{h}} + \tfrac{1}{2} a_z t_{\text{h}}^2 \Rightarrow z_{\text{noair}} = 1.942 \text{ ft.} \quad (16)$$

This is where an inexperienced batter might think the pitch will be if it had no spin.

Batters describe the effect of spin on the ball as the "break." One way to analytically define the break is the difference between where the ball actually arrives and where it would have arrived without any spin. The actual x- and z-positions are in Table I (items 12 and 13). So, this definition of break can now be calculated for the x- and z-directions.

$$x_{\text{break}} = x - x_{\text{noair}} = 0.723 \text{ ft} = 8.68 \text{ in.} \quad (17)$$

$$z_{\text{break}} = z - z_{\text{noair}} = 0.801 \text{ ft} = 9.61 \text{ in.} \quad (18)$$

These values are very close to the pfx_x and pfx_z values in Table I (items 10 and 11). This method of calculating the "break" of the pitch is somewhat arbitrary.[11] Other methods are possible and the last three pieces of data in Table I (items 23, 24, and 25) refer to a different method.[12]

Let's look at the forces involved in the motion of a major league pitch. Given the weight of a baseball is 0.320 lb, we can find the x-, y-, and z-components of the force exerted on the ball by the air during its flight. Since the components of the acceleration are given in Table I (items 20, 21, and 22), we can use Newton's second law along each direction. Along x and y the only force is due to the air,

$$F_x = ma_x = mg\left(\frac{a_x}{g}\right) = (0.320)\left(\frac{13.233}{32.174}\right) \quad (19)$$
$$= 0.132 \text{ lb}$$

$$F_y = ma_y = mg\left(\frac{a_y}{g}\right) = 0.257 \text{ lb.} \quad (20)$$

Along z, gravity is also in play,

$$F_z - mg = ma_z \Rightarrow F_z = mg\left(1 + \frac{a_z}{g}\right) = 0.146 \text{ lb.} \quad (21)$$

Batters often speak of a "rising fastball." Since this force is only about half the weight of the ball, the ball can't actually "rise" in the sense of moving vertically upward. However, it certainly will drop noticeably less than it would due to gravity alone. So, the "rising fastball" really only "rises" above the gravitational trajectory.

The magnitude of the force caused by the air is

$$F_{\text{air}} = \sqrt{F_x^2 + F_y^2 + F_z^2} = 0.324 \text{ lb} \ . \qquad (22)$$

The largest force is in the y-direction and is predominantly the drag of air resistance. The upward force of lift on the spinning ball in the z-direction is mostly created by the backspin about the x-axis while the force in the x direction is mostly from the sidespin about the z-axis. These forces due to the spin of the ball are related to the Magnus effect. It is amazing to realize that major league pitchers can spin a ball fast enough and give it sufficient velocity that the total force caused by the air is about the same as the weight of the ball.

The physics of pitching has been described many times before.[13-21] However, with the PITCHf/x data set, intricate studies of the flight of real pitches in actual game situations can be thoroughly examined. Alan Nathan's "The Physics of Baseball" website[22] is a great place to start if you want to get a sense of what intrepid souls are doing with PITCHf/x data. Now, when your students ask a question about the flight of baseballs, you can direct them to real life data from an actual pitch from their favorite pitcher or a pitch to their favorite hitter.

References

1. The author makes no apology for the use of English units throughout this paper. After all, they are the traditional units of the national pastime!
2. An amazingly sharp 30-frames/second video of Bonds hitting the ball can be found at http://www.usatoday.com/sports/graphics/bonds-756/flash.htm. Video of the entire at bat as well as just the last pitch can be found by searching Major League Baseball's site http://mlb.com.
3. http://www.sportvision.com is their homepage.
4. http://www.mlb.com is their home page. You may want to check out their Gameday feature that uses this data at http://mlb.mlb.com/mlb/gameday/.
5. Sportvision's description is at http://www.sportvision.com/main_frames/products/pitchfx.htm.
6. The server is located at http://gd2.mlb.com/components/game/mlb/.
7. http://webusers.npl.uiuc.edu/~a-nathan/pob/tracking.htm.
8. http://www.brooksbaseball.net/pfx/.
9. Ironically, just before this fateful pitch, the pitcher requested a new baseball. He tossed the old ball to the catcher. This event fooled the software into thinking the toss was an actual pitch. The data from this toss are what you will get if you use PITCHf/x Tool. MLB has put the corrected data on its server and that is where the author collected it.
10. The author has made little effort to track significant figures because the data from PITCHf/x has little regard for them. Just keep in mind that trajectory data is supposedly good to less than an inch.
11. Sportvision has chosen to compute the deviation of the ball from a "spinless trajectory" starting at 40 ft because the results are consistent with the thinking of experienced baseball people. A "physics definition" of break would likely start at the point where the pitcher releases the ball, but the resulting break would be much larger than baseball people are willing to accept.
12. See Ref. 6 for an explanation.
13. Alan M. Nathan, "The effect of spin on the flight of a baseball," *Am. J. Phys.* **76,** 119-124 (Feb. 2008).
14. Robert G. Watts and Ricardo Ferrer, "The lateral force on a spinning sphere: Aerodynamics of a curveball," *Am. J. Phys.* **55,** 40-44 (Jan. 1987).
15. Herman Erlichson, "Is a baseball a sand-roughened sphere?" *Am. J. Phys.* **53**, 582-583 (June 1985).
16. Cliff Frohlich, "Comments on 'Is a baseball a sand-roughened sphere?' " *Am. J. Phys.* **53,** 583 (June 1985).
17. Cliff Frohlich, "Aerodynamic drag crisis and its possible effect on the flight of baseballs," *Am. J. Phys.* **52,** 325-334 (April 1984).
18. Robert G. Watts and Eric Sawyer, "Aerodynamics of a knuckleball," *Am. J. Phys.* **43,** 960-963 (Nov. 1975).
19. Lyman J. Briggs, "Effect of spin and speed on the lateral deflection (curve) of a baseball; and the Magnus Effect for smooth spheres," *Am. J. Phys.* **27**, 589-596 (Nov. 1959).
20. Richard M. Sutton, "Baseballs do curve and drop!" *Am. J. Phys.* **10,** 201-202 (Aug. 1942).
21. Frank L. Verwiebe, "Does a baseball curve?" *Am. J. Phys.* **10,** 119-120 (April 1942).
22. See Ref. 6.

PACS codes: editor

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