Magnetically coupled rotors

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A system with intriguing oscillatory behavior is created using small magnets fixed to the edges of parallel rotating structures. Under certain conditions a system of two rotors will exchange velocities repeatedly in a manner similar to many coupled-oscillator systems, but without oscillations in their position. Simulation of a simple model consisting of two magnetic dipoles on a common axis of rotation yields results that closely match the experimental data.

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I. INTRODUCTION

Geomag kits consist of a number of magnetic rods and steel ball bearings. The bearings can be used as junctions between the rods, allowing multiple attachments at arbitrary angles to form geometric structures.[1] The magnets are strong enough to support fairly complex structures, even under tension, and one that particularly caught our attention was a hanging double rotor (see Fig. 1).

The rotors are suspended at a single contact point between hardened steel bearings, and as a result the friction is very low. If the angular velocities of the two rotors are similar, they will alternately exchange angular velocity. This behavior is similar to that of coupled oscillators, but the velocity oscillates rather than the position.

II. EXPERIMENTAL DATA

The Geomag coupled rotor assembly consists of 30 magnetic dipoles. We worked with a simpler arrangement of two magnetic dipoles on a common axis. To measure the motion of the dipoles, we used two PASCO rotary motion sensors attached to a Vernier LabPro interface. We mounted a small brass flywheel to each rotary motion sensor to increase the rotational inertia and proportionally decrease the effect of friction in the bearings. The magnetic dipole was supplied by gluing a disk-shaped neodymium magnet on the axis of each sensor, with the magnetic moment perpendicular to the axis. Both sensors were then mounted on a stand so that the rotational axes were colinear (see Fig. 2).

The data obtained by this method (see Fig. 3) clearly show oscillatory behavior. For the data set shown, rotor 1 was given an initial spin and rotor 2 was initially at rest. The velocity oscillations of rotor 2 increase in magnitude as the angular velocity of rotor 1 decreases until the system reaches the point where the two rotors have the same angular velocity. At this time, the two begin to exchange velocities. The decay is consistent with damping from a constant frictional torque.[2]

III. THEORY AND SIMULATION

The field of a magnetic dipole in coordinate-free form is given by[3]

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}[3(\mathbf{m}\cdot\hat{r})\hat{r} - \mathbf{m}], \tag{1}$$

where $\mathbf{m}$ is the magnetic moment of the dipole and $\hat{r}$ is a unit vector pointing from the dipole to the point at which the field is measured. The dipole moments are perpendicular to the common axis of rotation, so $\mathbf{m}\cdot\hat{r}=0$, and the magnetic field from dipole 1 at the position of dipole 2 is

$$\mathbf{B}_1 = -\frac{\mu_0\mathbf{m}_1}{4\pi r^3}. \tag{2}$$

The magnetic torque on dipole 2 due to dipole 1 is

$$\boldsymbol{\tau}_2 = \mathbf{m}_2 \times \mathbf{B}_1 = -\frac{\mu_0}{4\pi r^3}(\mathbf{m}_2 \times \mathbf{m}_1). \tag{3}$$

The two dipoles have the same magnitude $|\mathbf{m}|$, so the magnitude of the torque is

$$\tau_2 = -\frac{\mu_0}{4\pi r^3}m^2 \sin(\theta_1 - \theta_2) = I\ddot{\theta}_2, \tag{4}$$

where I is the rotational inertia. The angular acceleration is then

$$\ddot{\theta}_2 = -\beta \sin(\theta_1 - \theta_2), \tag{5}$$

where

$$\beta \equiv \frac{\mu_0 m^2}{4\pi r^3 I}. \tag{6}$$

To compare the model with the experimental apparatus, we add a constant frictional torque term $-b\dot{\theta}/|\dot{\theta}|$, and write the equations of motion for each rotor:

$$\ddot{\theta}_1 = -\beta \sin(\theta_2 - \theta_1) - b\frac{\dot{\theta}_1}{|\dot{\theta}_1|}, \tag{7a}$$

$$\ddot{\theta}_2 = -\beta \sin(\theta_1 - \theta_2) - b\frac{\dot{\theta}_2}{|\dot{\theta}_2|}. \tag{7b}$$

A closed-form solution to Eq. (7) is not available, but we can gain some insight into the problem by looking at the sum and difference of Eqs. (7a) and (7b). We define

$$\mathcal{S} \equiv \theta_1 + \theta_2, \tag{8}$$

$$\mathcal{D} \equiv \theta_1 - \theta_2. \tag{9}$$

If we add and subtract Eqs. (7a) and (7b), we obtain

$$\ddot{\mathcal{S}} = -b\left(\frac{\dot{\theta}_1}{|\dot{\theta}_1|} + \frac{\dot{\theta}_2}{|\dot{\theta}_2|}\right), \tag{10}$$

Fig. 1. Geomag coupled-rotor configuration.

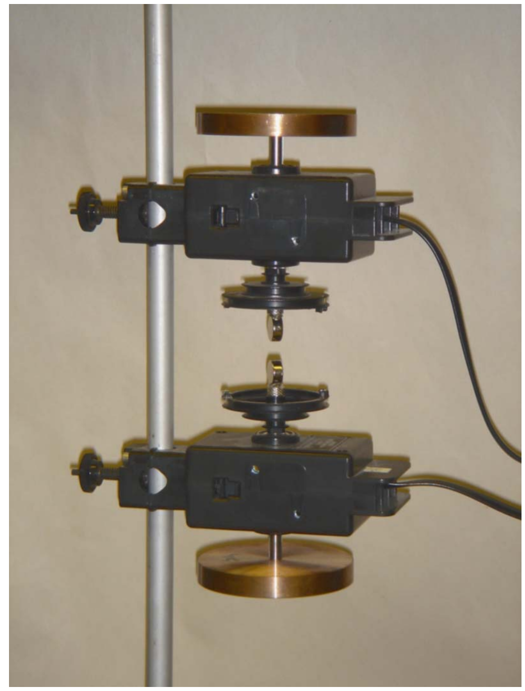

Fig. 2. Apparatus used for the experimental observations. Adjusting the spacing between the two rotors affects the strength of the interactions, but does not qualitatively change the behavior.

$$\ddot{\mathcal{D}} = 2\beta \sin \mathcal{D} - b\left(\frac{\dot{\theta}_1}{|\dot{\theta}_1|} - \frac{\dot{\theta}_2}{|\dot{\theta}_2|}\right). \tag{11}$$

Equation (11) with $b=0$ is the equation for the simple pendulum, with the coordinate system rotated so that the equilibrium position is at π instead of at 0. Hence, whatever the behavior of the individual rotors, the difference between the two rotors behaves similarly to a physical pendulum. Equation (10) with $b=0$ tells us that the total angular velocity and thus the angular momentum is conserved. For $b \neq 0$ we can see that the angular momentum decreases in a stepwise fashion: $\ddot{\mathcal{S}}=-2b$ if the signs of $\dot{\theta}_1$ and $\dot{\theta}_2$ are both positive, and $\ddot{\mathcal{S}}=0$ if one of $\dot{\theta}_1$ or $\dot{\theta}_2$ is negative.

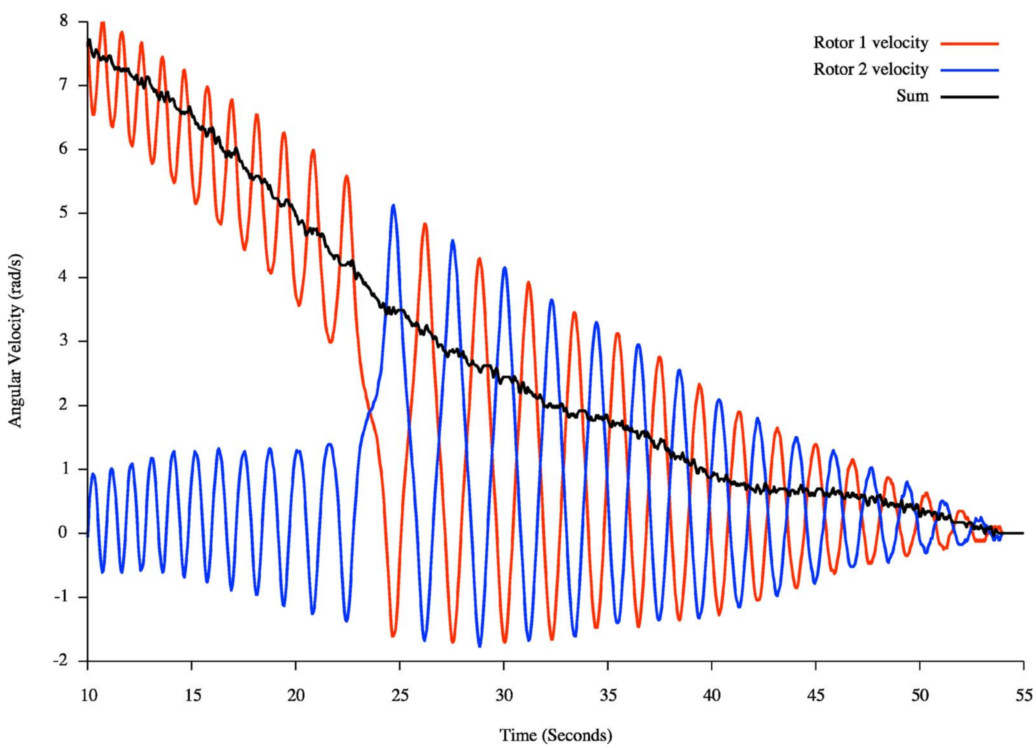

Fig. 3. Experimental data. Rotor 1 was given an initial angular velocity, rotor 2 was initially at rest.

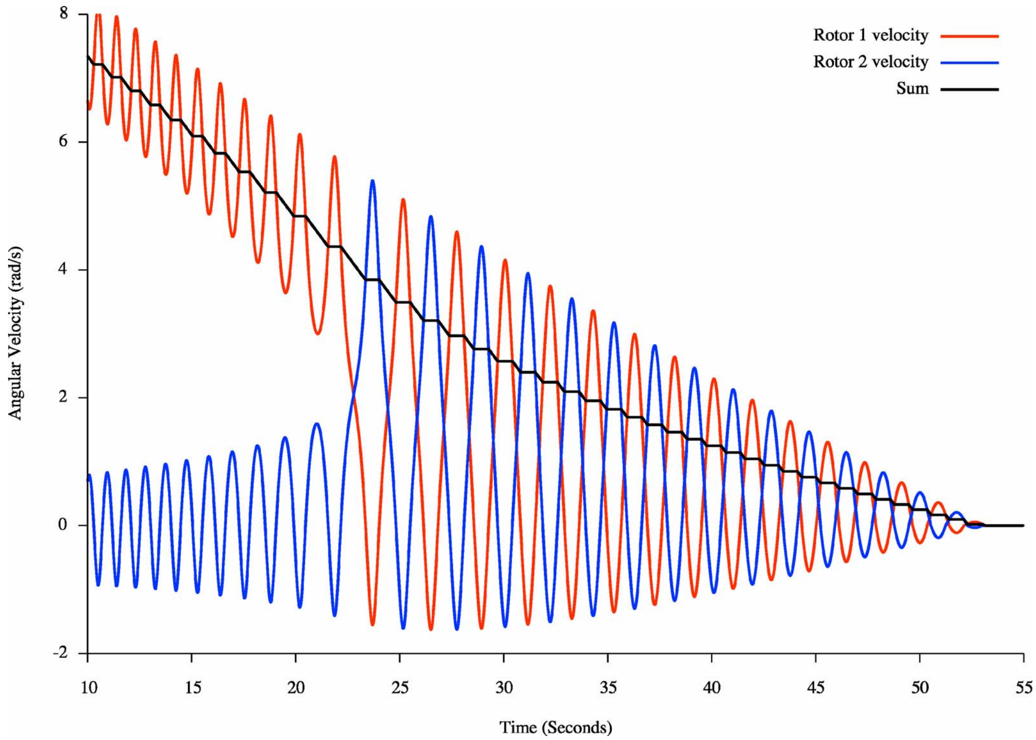

Fig. 4. Computed behavior of the system, showing the angular velocity of both rotors and the total velocity.

The equations of motion for this system lend themselves to a numerical solution. We used a fourth-order Runge-Kutta algorithm, with parameters β and b chosen to match our experimental data to obtain the results shown in Fig. 4. The stepwise decrease in the total angular velocity predicted by Eq. (10) is clearly visible. We can see hints of these steps in the sum in Fig. 3, particularly at $t \approx 25$ s, although these steps are at the limits of our experimental resolution.

IV. SUMMARY

When coupled oscillators are introduced in undergraduate physics courses, they are usually discussed in terms of oscillating positions. In the present apparatus, there is no common restoring force and the oscillations are in the velocity, rather than the position. The overall behavior of the system contains elements similar to the behavior of other well-known systems: damped simple harmonic motion and rotation with friction. The apparatus exhibits a wealth of interesting behavior and numerous conceptual links to other systems. For example, the factor-of-two change in the period of each rotor at the crossover point is analogous to the change in the period of a physical pendulum when it goes from looping around the axis to swinging back and forth. There is also an unexpected aspect of the theory, the stepwise decrease in the total angular momentum, which arises because when the rotors move in opposite directions, the frictional torques cancel and there is no net change in angular momentum. This stepwise decrease appears to be present in the experimental data. Despite the complexity of the behavior, the system can be modeled computationally without difficulty and the model matches the observed behavior closely.

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[1]W. C. K. Poon, “Two magnets and a ball bearing: A simple demonstration of the method of images,” Am. J. Phys. **71**, 943–947 (2003).

[2]John C. Simbach and Joseph Priest, “Another look at a damped physical pendulum,” Am. J. Phys. **73**, 1079–1080 (2005).

[3]David J. Griffiths, *Introduction to Electrodynamics* (Prentice Hall, Englewood Cliffs, NJ, 1999), 3rd ed., p. 246.