Nonphysical Results with the Electric-Field Mapping Experiment

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A common lab experiment used to introduce students to electric fields is the electric-field mapping experiment.[1] The apparatus consists of slightly conductive carbon-impregnated paper on which an arrangement of electrodes is drawn with conductive ink. (Or you may use a shallow pan of water with cut-out sheet-metal shapes.) When a power supply is attached to the electrodes, the paper acts as a voltage divider to the resulting current, allowing students to measure voltage differences between points on the paper with a voltmeter. By measuring the voltage difference between one of the conductors and points on the conductive paper, students can find equipotential lines. Then, by using the fact that the electric field must be perpendicular to the equipotentials, they can map out the electric field lines.

The voltage difference between two points on the paper is directly proportional to the electric current flowing between those two points. In many cases, this voltage difference is also proportional to the electrostatic potential that would be observed between those two points if the conductive paper were removed. It is important to remember, though, that the voltage being measured is due to a constant electric current distribution rather than a static distribution of charge. This distinction makes a considerable difference when we consider the behavior of the current near the boundary of the paper.

The difference arises because the electric current is confined to the paper, but the expected electric field is not. At first glance, we would expect to find a pattern of equipotential lines similar to those shown in most textbooks, for which the potential is defined to be zero at large distances from the conductors. Figure 1 shows such equipotentials for a typical "parallel-plates" arrangement of conductors, for which the two-dimensional system consists only of two parallel plates.[2] The equipotentials actually found are quite different. At the edge of the paper, the current must run parallel to the edge. Since the current is parallel to the edge, the "equipotential lines" found with the voltmeter will be perpendicular to the edge of the paper.

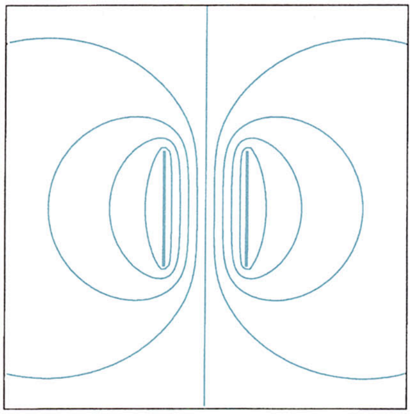

Fig. 1. Equipotential lines for oppositely charged electrodes surrounded by infinite empty space.

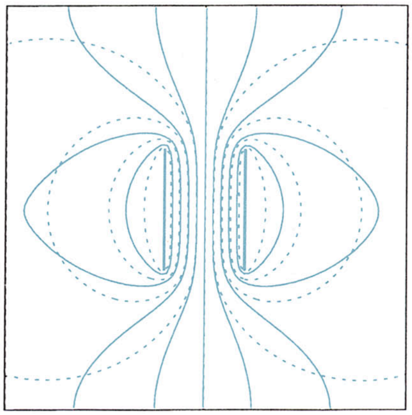

Fig. 2. Nonphysical "equipotential lines" obtained with electric-field mapping apparatus used without a conductive boundary. Dotted lines show the "expected" results (Fig. 1) for comparison.

The most common approach to solving this type of problem computationally is the relaxation method. In its simplest application, the area of interest is divided into a grid of points, with the value at each point on the grid representing the electric potential at that point. "Fixed" points, corresponding to points held at a constant potential, are assigned their respective values and marked as fixed; the other points on the grid are assigned values based on an initial guess of what the potential looks like. (The initial guess is not critical; a good guess will decrease calculation time, but any guess—including zero—will result in the same solution.) The relaxation routine scans through the grid and changes each nonfixed point to the average of the surrounding points. It also keeps track of the largest change made in a single pass. This process of averaging each point is repeated until the largest change made is less than the desired level of accuracy. This method is very reliable, although quite slow. (The calculations in this paper were made with a faster variation of the same method, called Successive Over-Relaxation.[3])

The program written for this investigation used a 100×100 grid of points and calculated the potential at each interior point as being the average of the nearest eight points. The boundary points were calculated by letting each point on an edge equal the average of the nearest *five* points (or *three* points for the corners), which ensures that the gradient at an edge be parallel to the edge.

The resulting equipotential lines, which have been verified qualitatively by students in our introductory labs, are shown in Fig. 2. (The dotted lines show the expected result from Fig. 1 for comparison.) Not only is this pattern different from what is expected, but there is no simple two-dimensional electrode shape that will give such a pattern.[4]

Since the difference between the real and expected result is due to a poorly modeled boundary condition, it is best to fix the boundary. The easiest way to accomplish this is by adding a border of conductive ink to the paper and holding this boundary at a fixed potential. (If you are using trays of water, an equivalent solution is to use a bare metal tray with a flat plastic sheet in the bottom.) We would expect the electric field to be perpendicular to this boundary, since it's a good conductor. The measured electric current will also be perpendicular to the boundary; so the equipotential lines found will be parallel to the boundary at the boundary.

This boundary condition can be modeled computationally with a few slight changes to the relaxation program written previously. Instead of having the program use a special routine to calculate the potential for the edge points, we define the edges to be fixed at some set value (zero in our case) and then don't include the edge points in the relaxation process. This modification gives the equipotential lines shown in Fig. 3, which are just as we would expect them to be for this set of electrodes. (Again, this solution has been qualitatively verified by students in our introductory lab.) These equipotential lines differ from the original expectation (Fig. 1), but they have a significant advantage over Fig. 2 in that they are actual solutions to an electrostatic problem: charged conductive plates inside a two-dimensional box.

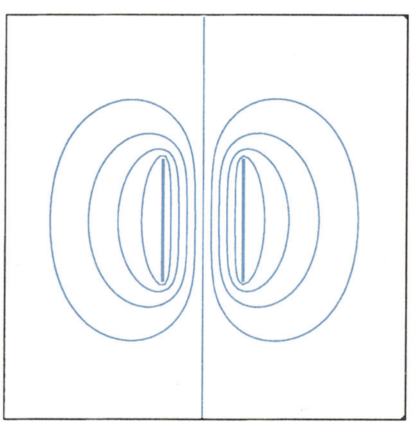

Fig. 3. Equipotential lines found when the apparatus is used with a conductive boundary. This pattern is expected for oppositely charged electrodes in a conductive box.

Summary

To obtain solutions with this electric-field mapping apparatus that correspond to real electrostatic problems, it is necessary to explicitly define the boundary conditions. Charge configurations with boundary conditions "at infinity" can lead to unexpected and incorrect results.

Acknowledgments

I would like to thank Robert Beichner and Richard Mowat for their help and encouragement.

References

1. Available from most lab equipment suppliers; for example, PASCO scientific's PK-9023.
2. The equipotential lines here are calculated directly from the charge distribution in the conductors.
3. M. DiStasio and W.C. McHarris, "Electrostatics Problems? Relax!" *Am. J. Phys.* **42** (9), 792–793 (September 1974).
4. It may be possible to reproduce this pattern with a distribution of charge *outside* of the measurement region, but the methods required to calculate such a distribution are beyond the scope of an introductory physics course.