Rüchardt's Measurement of γ using a Ground-Glass Syringe

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Rüchardt's Method

One common method of measuring the ratio of heat capacities of gases, $\gamma = \frac{C_p}{C_v}$, is Rüchardt's method. This method uses the oscillation of a mass supported by the pressure of the gas.

Traditionally, this is done with a steel bearing oscillating in a precisely-fitted glass tube attached to a gas reservoir. Exact measurements of the frequency are difficult by this method, however, so we will be using a ground-glass syringe[1] and magnetic pickup instead.

Mathematical Background

For an adiabatic process,

$$PV^{\gamma} = \text{ constant} \equiv K \ .$$

We sum the forces on the system:

$$PA - P_0A - mg = \frac{K}{V^{\gamma}}A - P_0A - mg = ma \tag{1}$$

where at the equilibrium position, $ma = 0$.

We will take small oscillations δx about the equilibrium position x, noting that $V = A(x + \delta x)$, and expand the first term in equation 1 using the binomial expansion:[1]

$$\frac{KA}{V^{\gamma}} = \frac{KA}{A^{\gamma}(x+\delta x)^{\gamma}} = \frac{KA}{(Ax)^{\gamma}\left(1+\frac{\delta x}{x}\right)^{\gamma}} \approx \frac{KA}{V_0^{\gamma}}\left(1-\gamma\frac{\delta x}{x}\right) \ . \tag{2}$$

We note that the equilibrium pressure in the syringe is

$$P_e \equiv \frac{K}{V_0^{\gamma}} \ .$$

Substitution of this definition and equation 2 into equation 1 gives us

$$P_eA\left(1-\gamma\frac{\delta x}{x}\right) - P_0A - mg = ma$$

$$P_eA - \frac{P_eA\gamma\delta x}{x} - P_0A - mg = ma$$

[1] Thanks to Matt Smith for this improved derivation.

but our equilibrium condition states that

$$P_e A - P_0 A - mg = 0 \ ,$$

so

$$ma = -\frac{P_e A \gamma \delta x}{x}$$

or since $x = V/A$,

$$a = -\left[\frac{\gamma A^2 P_e}{mV}\right] \delta x \ . \tag{3}$$

Equation 3 is the equation for simple harmonic motion, with ω^2 equal to the term in brackets. This gives us an equation relating γ to measurable quantities:

$$f_o = \frac{\omega}{2\pi} = \frac{A}{2\pi}\sqrt{\frac{\gamma P_e}{mV}} \ . \tag{4}$$

One thing to note is that equation 4 is for undamped motion. The plunger is damped by the viscosity of the gas in the syringe, so the measured frequency f will be lower than f_o. For under-damped simple harmonic motion with viscous damping, such as for this system, we can expect that the excursion X of successive maxima on one side of the oscillation would go as

$$X = X_o e^{-\lambda n} \tag{5}$$

where n is some oscillation number. In this case, the measured frequency f is related to the undamped frequency f_o by[2]

$$\frac{f_o}{f} = \sqrt{1 + \left(\frac{\lambda}{2\pi}\right)^2} \ . \tag{6}$$

Procedure

1. Make sure the syringe is well-braced within its support structure.

2. Measure all relevant experimental parameters. When measuring the mass of the plunger, please do not touch the ground-glass portion of the apparatus or the inside of the syringe. A layer of finger oil is sufficient to cause significant friction!

   When considering the equilibrium pressure P_e in the syringe, remember that the pressure is not just air pressure.

[2]Please note that the equivalent equation in [1] is erroneous. See [2] or [3].

3. Remove the cap from the bottom of the syringe by twisting gently. Adjust the syringe to the desired gas volume and replace the cap. Note that the volume will decrease gradually over time due to leakage past the plunger, so be sure to record the volume that was actually used, rather than the volume you set the apparatus to fifteen minutes before you took the data!

4. Adjust the coil support so that the magnet at the top of the plunger is centered on, and just at the bottom of, the coil.

5. Set the digital oscilloscope so that it is prepared to save a single sweep of the coil signal. "Pluck" the plunger lightly. You may have to try this a few times to get a good signal on the 'scope.

6. Use the cursors on the 'scope to measure and record amplitude and time values for as many successive peaks as possible.

7. Calculate γ. I would suggest using curve fits to calculate f and λ from the data. Be sure to include estimates of uncertainty in your results!

Repeat the measurement for several gases. Supplies may vary, but we typically have helium, argon, carbon dioxide, nitrogen, and methane available somewhere around the department.

Safety note

The syringe and plunger are delicate, they are a matched set, and they are expensive. Be gentle! Do *not* fill the syringe directly from a high-pressure source with the plunger in place, as it is difficult to retrieve the plunger from the ceiling intact.

Instructor's Notes

Equipment Sources

Ground-glass syringe Fisher Scientific, or other biological supply house.

`http://tinyurl.com/2qdfjv`

I recommend at least 50 cc, and a larger one is preferred.

Magnet Wondermagnet.com is a wonderful resource for all sorts of Neodymium magnets.

`http://wondermagnet.com`

The exact size and shape really doesn't matter much – it should just fit in your coil.

Coil Any coil will do. The one I use was obtained from a pile of scrap electronics parts in the back of the electronics shop. Other good sources include disassembling a tweeter speaker from a garage sale, or a pair of old headphones, or even making one of your own by wrapping a hundred turns of fine wire around a sewing-machine bobbin.

Base Find a board, drill a hole to match your syringe.

Experiment Notes

1. The correction for damped/undamped frequency (equations 4.5 and 4.6, and related discussion) makes almost no difference at all. The size of the correction offered by these equations is typically an order of magnitude smaller than the cumulative error for the rest of the experiment. My reason for leaving this part of the lab in place is curiosity, mostly: I like to see which students figure out that it's not important. (In three years of doing the lab, I've had one student point out that it was negligible!)

2. Student results are typically within 5-15% of accepted values, usually on the low side.

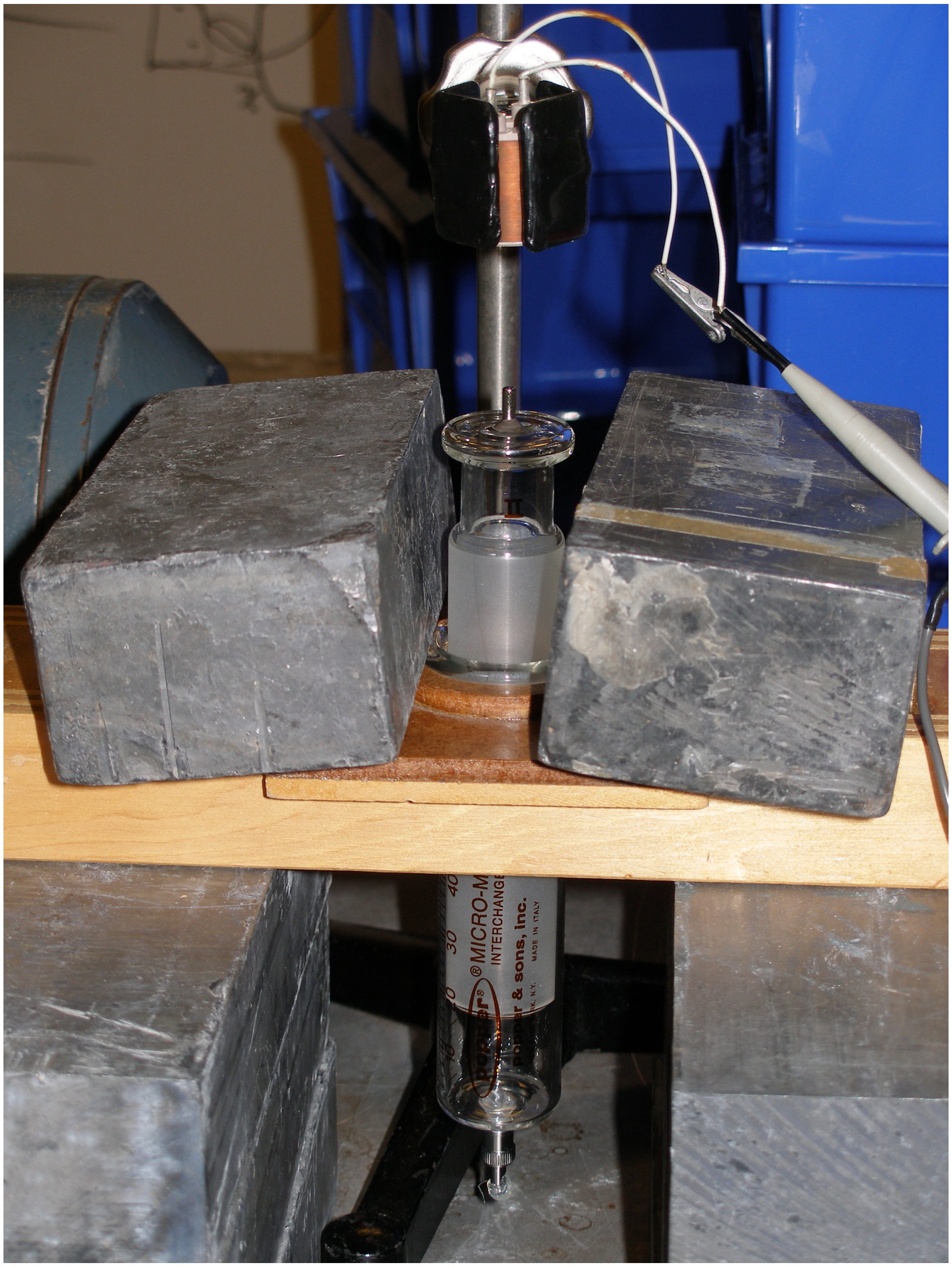

Figure 1: Photo of experimental apparatus

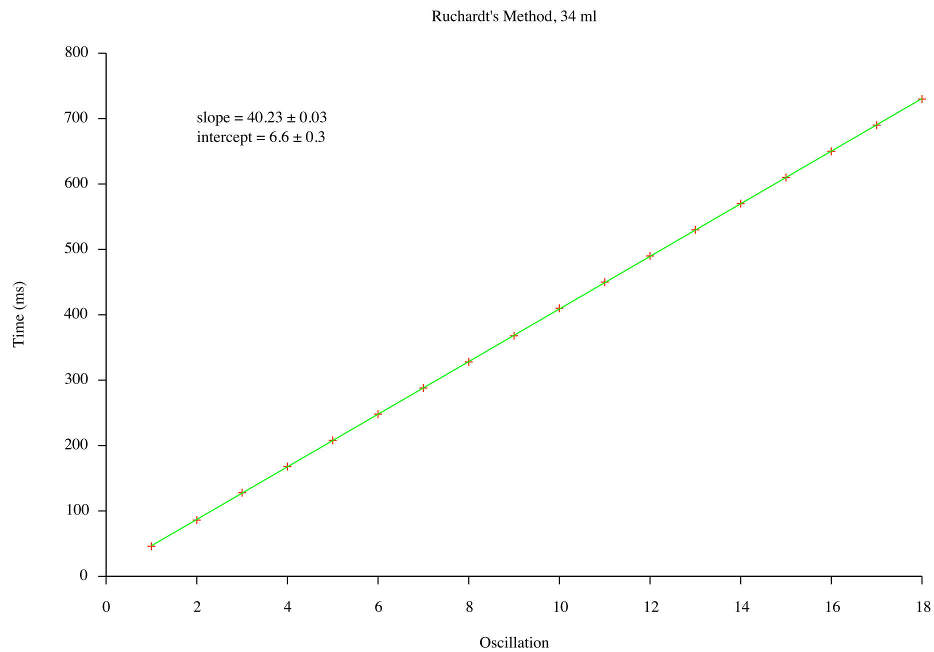

Figure 2: Plot of time v. oscillation number, used to accurately determine the frequency of oscillation of the syringe plunger.

Bibliography

[1] J. L. Hunt. Accurate experiment for measuring the ratio of specific heats of gases using an accelerometer. *American Journal of Physics*, 53(7):696–697, July 1985.

[2] J. B. Marion and S. T. Thornton. *Classical Dynamics*. Harcourt, Brace, Jovanovich, 3 edition, 1988.

[3] H. Lamb. *Dynamics*. Cambridge University Press, 2nd edition, 1961.